$B$ and $D$ mesons on the lattice

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High Energy Physics
Illinois

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Outline

1. Introduction: Lattice QCD

2. Decay constants: $P \to l\nu$
   - $f_D$ and $f_{D_s}$: test of lattice QCD
   - $f_B$ and $f_{B_s}$

3. Semileptonic decays
   - Exclusive $B \to D^*l\nu$: determination of $|V_{cb}|$
   - $B \to \pi l\nu$: determination of $|V_{ub}|$

4. $B^0 - \bar{B}^0$ mixing: $\Delta M_{d,s}$, $\Delta \Gamma_{d,s}$ and $\xi$

5. Conclusions and outlook
1. Introduction: Lattice QCD

Lattice QCD is a quantitative non-perturbative formulation of QCD based only on first principles.

It provides a quantitative calculation tool, becoming a precise tool.
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# It provides a quantitative calculation tool → becoming a *precise tool*

# Precise lattice calculations: for stable (or almost stable) hadrons, masses and amplitudes with no more then one initial (final) state hadron.

* Quantities relevant for all CKM matrix elements except $V_{tb}$.

\[
\text{experiment} = (\text{CKM})^*(\text{lattice inputs})
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Lattice inputs: Encoding non-perturbative information on hadrons

(decay constants, form factors, bag parameters, etc)
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* Generate sets of gluon fields contribute most to the Path Integral (configurations).

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Goal: **control systematic errors**
Quenched approximation: neglect vacuum polarization effects → uncontrolled and irreducible errors

Unquenched work with $N_f = 2 + 1$ flavours of sea quarks
**Quenched approximation**: neglect vacuum polarization effects

→ uncontrolled and irreducible errors

Unquenched work with \( N_f = 2 + 1 \) flavours of sea quarks

\[ N_f = 2 \text{ and } N_f = 2 + 1 \] ensembles available

# \( m_l > m_{u,d} \) in numerical simulations
Quenched approximation: neglect vacuum polarization effects → uncontrolled and irreducible errors

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$N_f = 2$ and $N_f = 2 + 1$ ensembles available

# $m_l > m_{u,d}$ in numerical simulations

Use chiral perturbation theory to extrapolate to $m_{u,d}$
Testing Lattice QCD

$N_f = 0$

- $f_\pi$
- $f_K$
- $m_\Omega$
- $m_N$
- $m_{D_s}$
- $m_D$
- $m_{D_s}^* - m_{D_s}$
- $m_\psi - m_{\eta_c}$
- $\psi(1P-1S)$
- $2m_{B_{s,av}} - m_\Upsilon$
- $m_{B_c}$
- $\Upsilon(3S-1S)$
- $\Upsilon(2P-1S)$
- $\Upsilon(1P-1S)$
- $\Upsilon(1D-1S)$

$N_f = 2 + 1$

- $m^l_{D_s}$ = 1.962(6)
- $m^e_{D_s}$ = 1.968
- $m^l_{D}$ = 1.868(7)
- $m^e_{D}$ = 1.868

Experimental quantities are quite well reproduced by lattice
when including realistic sea quark effects
# Purely leptonic decays can be used to extract CKM matrix elements

\[ \Gamma(P_{ab} \rightarrow l\nu) \propto f_P^2 |V_{ab}|^2 \]

or testing SM/lattice predictions
$f_D$ and $f_{D_s}$: test of lattice QCD

\[ B(D_q \rightarrow l\nu) \propto |V_{cq}|^2 \cdot f_{D_q}^2 \]

experiment lattice

# Simple matrix element

\[ \langle 0 | \bar{q} \gamma_\mu \gamma_5 c | D_q(p) \rangle = i f_{D_q} p_\mu \rightarrow \text{precise calculations} \]
$f_D$ and $f_{D_s}$: test of lattice QCD

$$B(D_q \to l\nu) \propto |V_{cq}|^2 f_{D_q}^2$$

- Simple matrix element $\langle 0|\bar{q}\gamma_\mu\gamma_5 c|D_q(p)\rangle = i f_{D_q} p_\mu \rightarrow$ precise calculations
- Results from two groups with $N_f = 2 + 1$
  - Heavy valence quarks HPQCD, HISQ, FNAL/MILC Fermilab action

- Highly improved staggered quarks (HISQ): Reduction of $O(a^2 \alpha_s)$ and $O((am_Q)^4)$ discretization errors $\rightarrow$ Very precise results for charm physics, charmonium and D, ($m_c$ fixed by $\eta_c$). E. Follana et al (2007)
$f_D$ and $f_{D_s}$: test of lattice QCD

$B(D_q \to l\nu) \propto |V_{cq}|^2 f_{D_q}^2$

experiment \hspace{1cm} lattice

Simple matrix element $\langle 0 | \bar{q} \gamma_\mu \gamma_5 c | D_q(p) \rangle = i f_{D_q} p_\mu \to$ precise calculations

Results from two groups with $N_f = 2 + 1$

- Heavy valence quarks \textbf{HPQCD} \hspace{3mm} \textbf{HISQ} \hspace{3mm} \textbf{FNAL/MILC} \hspace{3mm} \textbf{Fermilab action}

- \textbf{MILC} ensembles: 3 lattice spacings (0.09 fm, 0.12 fm, 0.15 fm)

- Renormalization partially non-pert. (\textbf{FNAL/MILC}, 1.5% error) and normalization via PCAC (\textbf{HPQCD}, no error)

- Simultaneous chiral and continuum extrapolation including all $a$, valence and sea quark masses:
  - \textbf{SChPT} (\textbf{FNAL/MILC}) and continuum ChPT + $O(a^2)$ terms (\textbf{HPQCD}).

- Highly improved staggered quarks (\textbf{HISQ}): Reduction of $O(a^2 \alpha_s)$ and $O((am_Q)^4)$ discretization errors $\to$ Very precise results for charm physics, charmonium and $D$, ($m_c$ fixed by $\eta_c$). E. Follana et al (2007)\textbf{HPQCD}
# Sensitive to $\textbf{BSM}$ physics: Starting to see evidence for nonstandard leptonic decays of $D_s$ mesons?  
Dobrescu and Kronfeld 2008
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# $> 3\sigma$ discrepancy between experiment and **HPQCD** lattice $f_{D_s}$. 
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$\#$ Experiment-lattice agreement in $f_K$, $f_\pi$, $f_D$, $m_D$, $m_{D_s}$, $\frac{2m_{D_s} - m_{\eta_c}}{2m_D - m_{\eta_c}}$. 
# Sensitive to **BSM** physics: Starting to see evidence for nonstandard leptonic decays of $D_s$ mesons? Dobrescu and Kronfeld 2008

![Graph showing $f_D$ and $f_{D_s}$](image)

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# Expected reduction of experimental errors

# Experiment uses $V_{cs} = V_{ud}$. 


\[ f_B \text{ and } f_{B_s} \]

# Extraction of CKM matrix elements:

\[ B(B^− \rightarrow τ^− \bar{ν}_τ) \propto |V_{ub}|^2 \]

\[ f_B^2 \hspace{1cm} \text{experiment} \]

\[ \langle 0|\bar{q}\gamma_\mu \gamma_5 b|B_q(p)\rangle = i f_{B_q} p_\mu \]

\[ \text{lattice} \]
$f_B$ and $f_{B_s}$

**Extraction of CKM matrix elements:**

\[
B(B^- \to \tau^- \bar{\nu}_\tau) \propto |V_{ub}|^2 \frac{f_B^2}{f_{B_s}^2}
\]

Experiment

Lattice

\[
\langle 0| \bar{q} \gamma_\mu \gamma_5 b|B_q(p) \rangle = i f_{B_q} p_\mu
\]

**Decay constants needed in the SM prediction for processes potentially very sensitive to BSM effects:** for example, $f_{B_s}$ for $B_s \to \mu^+ \mu^-$

$B^- \to \tau^- \bar{\nu}_\tau$ is a sensitive probe of effects from charged Higgs bosons.
# Extraction of CKM matrix elements:

\[ B(B^- \to \tau^- \bar{\nu}_\tau) \propto |V_{ub}|^2 \]

experiment \hspace{1cm} \text{lattice} \hspace{1cm} f_B^2

\[ N_f = 2 + 1 \text{ determinations} \]

heavy valence quarks HPQCD \hspace{1cm} \text{NRQCD} \hspace{1cm} \text{FNAL/MILC} \hspace{1cm} \text{Fermilab action}

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Heavy valence quarks HPQCD, NRQCD, FNAL/MILC Fermilab action

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<td>( f_B ) (MeV)</td>
<td>197 ± 13</td>
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<td>6.8-10.3</td>
<td>4.0</td>
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J. Shigemitsu 2007

Extraction of \( f_{B_s}/f_B \) from double ratios: e.g. \([f_{B_s}/f_B]/[f_K/f_\pi]\)
3. Semileptonic decays

\[ J = V_{\mu}, A_{\mu} \]
\[ V_{ij} \]

\[ W \]
\[ \nu \]

\[ \mu \]

\[ P_1 \]
\[ P_2 \]
Exclusive $B \to D^* l \nu$: determination of $|V_{cb}|$

- $B \to D^* l \nu$ rate at zero recoil $\propto |V_{cb} h_A(1)|$

- $|V_{cb}|$ needed as an input in $\epsilon_K$ and rare kaon decays ($Br(K \to \pi \nu \bar{\nu})$).
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- New double ratio method: $|h_A(1)|^2 = \frac{\langle D^* | \bar{c} \gamma j \gamma 5 b | \bar{B} \rangle \langle \bar{B} | \bar{b} \gamma_j \gamma 5 c | D^* \rangle}{\langle D^* | \bar{c} \gamma 4 c | D^* \rangle \langle \bar{B} | b \gamma 4 b | \bar{B} \rangle}$

$N_f = 2 + 1$ FNAL-MILC (Laiho, 2008)
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$N_f = 2 + 1$  

FNAL-MILC (Laiho, 2008)

\[ h_{A_1}(1) = 0.921(13)^{\text{stat.}}(19)^{\text{syst.}} \]

$|V_{cb}| \times 10^3 = 38.8(0.6)^{\text{exp.}}(1.0)^{\text{latt.}}$
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(one year)
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(one year)

Inclusive determination is $|V_{cb}| \times 10^3 = 41.7(0.7) \ (2\sigma \text{ difference})$
$B \to \pi l\nu$: determination of $|V_{ub}|$

Only $N_f = 2 + 1$ calculation so far: staggered HPQCD PRD73/75 (2006/07)

$$Br(B \to \pi l\nu) = |V_{ub}|^2 \int_0^{q_{max}^2} dq^2 f_{B\to\pi}(q^2)^2 \times \text{(known factors)}$$

NRQCD for $b$ valence quarks
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NRQCD for $b$ valence quarks

$|V_{ub}| \times 10^3 = 3.55(25)_{\text{exp.}}(50)_{\text{theor.}}$

14% theory error dominated by statistics and matching
**B → πlν: determination of |V_{ub}|**

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**NRQCD** for $b$ valence quarks

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# Poor overlap in $q^2$ between lattice and experiment
→ increases the total error
* Moving NRQCD: Generate data at low $q^2$ + keeping statistical errors under control K. Wong Lattice2007.
Work in progress to reduce total error

* **Moving NRQCD**: Generate data at low $q^2$ + keeping statistical errors under control [K. Wong Lattice2007].

* **z-fit**: combine lattice and experimental data over full $q^2$ region using model-independent expression based on analyticity and unitarity [Arnesen et al.; Becher & Hill; P. Ball; P. Mackenzie and R. Van de Water]

Simultaneous fit of lattice and BABAR $F_+$ data

$\chi^2$/d.o.f. = 0.4; C.L. = 0.99

- 3 parameter z-fit: $a_1/a_0 = -1.16 +/- 0.06; a_2/a_0 = -3.23 +/- 0.30$
- sys. error from rSχPT fit w/ additional NNLO analytic terms
- "best fit" of $f_{p+}, f_{\perp}$ w/ $g_\pi = 0.27$
- 12-bin BABAR rescaled by $|V_{ub}|$ from 3-parameter fit

# Work in progress to reduce total error

* Moving NRQCD: Generate data at low $q^2$ + keeping statistical errors under control K. Wong Lattice2007.

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Simultaneous fit of lattice and BABAR $F^+$ data

$\chi^2$/d.o.f. = 0.4; C.L. = 0.99


# Work underway to analyze systematics → FNAL-MILC (Mackenzie, LAT07)

total error after finishing current analysis ∼12%.
Semileptonic decays: Improvements in progress

\# \( D \rightarrow \pi l \nu \) and \( D \rightarrow K l \nu \):

**FNAL-MILC** working on \( N_f = 2 + 1 \) improvement of 2005 calculation of the form factors \( f_{+}^{D \rightarrow \pi}(0) \) and \( f_{+}^{D \rightarrow K}(0) \) (reduction of discr. errors)

\[ \rightarrow V_{cd} \text{ and } V_{cs}. \]
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$D \to \pi l \nu$ and $D \to K l \nu$:

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$\to V_{cd}$ and $V_{cs}$.

* Becirevic, Haas and Mescia: Testing systematic errors reduction for several double ratios with $N_f = 2$ Wilson fermions.
Semileptonic decays: Improvements in progress

\[ D \rightarrow \pi l \nu \text{ and } D \rightarrow Kl \nu: \]

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* Becirevic, Haas and Mescia: Testing systematic errors reduction for several double ratios with \( N_f = 2 \) Wilson fermions.

* \( \frac{\Gamma(D \rightarrow l \nu)}{\Gamma(D \rightarrow \pi l \nu)} \) independent of \( |V_{cq}| \rightarrow \) consistency check

* \( \frac{\Gamma(D_s \rightarrow l \nu)}{\Gamma(D \rightarrow Kl \nu)} \) CKM independent test of lattice (QCD)
# $B \to Dl\nu$ (alternative determination of $V_{cb}$):

de Divitiis et al 2007 Quenched analysis

* Including the case of non-vanishing lepton mass.

** Can study $Br(B \to D\tau\nu_\tau)/Br(B \to Dev_e)$, which is a good place to look for charged Higgs contributions to low energy observables.


** Lepton-flavour universality checks on the extraction of $V_{cb}$ are possible.
4. $B^0 - \bar{B}^0$ mixing: $\Delta M_{d,s}$, $\Delta \Gamma_{d,s}$ and $\xi$

### Experimental measurements:

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- theoretically: In the Standard Model

\[ \Delta M_q|_{\text{theor.}} \propto |V_{tq}^* V_{tb}|^2 \quad f_B^2 \hat{B}_B q \]

$\implies$ Need accurate theoretical calculation of $f_B^2 \hat{B}_B q$
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$\implies$ Need accurate theoretical calculation of $f_{Bq}^2 \hat{B}_{Bq}$

Precise determination of CKM matrix elements

$$\left| \frac{V_{td}}{V_{ts}} \right| = \frac{f_{Bs} \sqrt{B_{B_s}}}{f_{Bd} \sqrt{B_{B_d}}} \sqrt{\frac{\Delta M_d M_{B_s}}{\Delta M_s M_{B_d}}} \sqrt{\frac{\Delta M_d}{\Delta M_s}} \sqrt{\frac{M_{B_s}}{M_{B_d}}} \sqrt{\xi}$$

* Many uncertainties in the theoretical (lattice) determination cancel totally or partially in the ratio
NP could enter through new particles in box diagrams.

Recent claims of NP effects in the $B_s^0 - \bar{B}_s^0$ and $B_d^0 - \bar{B}_d^0$ systems
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# Two unquenched $N_f = 2 + 1$ calculations underway: HPQCD and MILC/FNAL

* Improved staggered (Asqtad) for light quarks and NRQCD ( HPQCD ) Fermilab action ( MILC/FNAL )

* Calculation of all the matrix elements needed to determine $\Delta M_{d,s}$, $\Delta \Gamma_{d,s}$ and $\xi$. 
NP could enter through new particles in box diagrams.

Recent claims of NP effects in the $B^0_s - \bar{B}^0_s$ and $B^0_d - \bar{B}^0_d$ systems (Bona et al. (UTfit Col.), arXiv:0803.0659; Lunghi and Soni, arXiv:0803.0512; Buras and Guadagnoli, arXiv:0805.3887)

Two unquenched $N_f = 2 + 1$ calculations underway: HPQCD and MILC/FNAL

- Improved staggered (Asqtad) for light quarks and NRQCD (HPQCD) Fermilab action (MILC/FNAL)
- Calculation of all the matrix elements needed to determine $\Delta M_{d,s}$, $\Delta \Gamma_{d,s}$ and $\xi$.

Current status: working on the chiral extrapolation (NLO+analytic NNLO $\chi$PT)
Preliminary results for $f_{B_q} \sqrt{M_{B_q} B_{B_q}}$

$$f_{B_s} \sqrt{M_{B_s} \hat{B}_{B_s}}(\text{GeV}^{3/2})$$

with $m_{s}^{valence}$ fixed to its physical value and $m_{s}^{sea}$ very close to it.

Statistics + fitting errors $\sim 1 - 2\%$

Statistics and systematic errors included

Same for $f_{B_d} \sqrt{B_{B_d}}$
Preliminary results for \( f_{Bq} \sqrt{M_{Bq} \hat{B}_{Bq}} \)

\[
f_{Bq} \sqrt{M_{Bq} \hat{B}_{Bq}} (\text{GeV}^{3/2})
\]

Fermilab/MILC

Example: Ensembles with \( a = 0.12 \text{ fm} \).

Full QCD: only statistical errors included.
Preliminary results for $\xi$: Full QCD

\[ \xi \frac{M_{B_s}}{M_{B_d}} = \left( f_{B_s} \sqrt{M_{B_s} B_{B_s}} \right) / \left( f_{B_d} \sqrt{M_{B_d} B_{B_d}} \right) \]

# Only statistical errors included.

# Only full QCD points included.
## Discussion of errors

<table>
<thead>
<tr>
<th>Term</th>
<th>Estimate (as %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{B_q} \sqrt{B_{B_q}}$</td>
<td>5 – 7%</td>
</tr>
<tr>
<td>$\xi$</td>
<td>2 – 3%</td>
</tr>
<tr>
<td><strong>Total (estimate)</strong></td>
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</table>
# Discussion of errors

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## Expected improvements in 2 years:
- smaller lattice spacings,
- better statistics,
- development of non-perturbative or partially non-perturbative matching,
- more accurate inputs ($a m_b$, $a$, ...).

## Reduction of errors by a factor of 1.5 – 2
Discussion of errors

\[ f_{Bq} \sqrt{B_{Bq}} \xi \]

| Total (estimate) | \(5 - 7\%\) | \(2 - 3\%\) |

# Expected improvements in 2 years: smaller lattice spacings, better statistics, development of non-perturbative or partially non-perturbative matching, more accurate inputs (\(a m_b, a, \ldots\)).

**Reduction of errors by a factor of 1.5 – 2**

# Underway RBC/UKQCD: C. Albertus et al.

* In an early stage: static limit, \(m_{\text{pion}} \geq 400\text{MeV}, \ldots\)
$B^0$ and $D^0$ mixing beyond the SM

# Effects of heavy new particles seen in the form of effective operators built with SM degrees of freedom (short-distance contributions for $D^0 - \bar{D}^0$)

$$\mathcal{H}_{eff}^{\Delta F = 2} = \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3} \tilde{C}_i \tilde{Q}_i$$

** With $Q_i$ and $\tilde{Q}_i$ four-fermion operators
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- $C_i, \tilde{C}_i$ Wilson coeff. calculated for a particular BSM theory
- $\langle F^0|Q_i|F^0 \rangle$ calculated on the lattice

SM predictions $+$ BSM contributions $+$ experiment $\rightarrow$ constraints on BSM physics
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# Same programme can be applied for extra operators
**B^0 and D^0 mixing beyond the SM**

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\[
\mathcal{H}_{eff}^{\Delta F=2} = \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3} \tilde{C}_i \tilde{Q}_i
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**Same programme can be applied for extra operators**

**Complete N_f = 2 + 1 analysis of \Delta B = 2 matrix elements expected from both FNAL-MILC and HPQCD, and \Delta D = 2 from FNAL-MILC in 1-2 years with errors < 10%**.
5. Conclusions and outlook

# Important progress in lattice calculations including sea quarks
\((N_f = 2 + 1)\)

* Precise new results (few percent errors) in \(D\) sectors.

* Expected for this year: precise results in \(b\) physics: \(B^0\) mixing parameters, decay constants.
5. Conclusions and outlook

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# Prospects for next two years

* Reduction in uncertainties of quantities relevant for CKM physics by a factor of around 2.

* Consistency checks of lattice QCD methods by …

  ** more comparison against experiment.

  ** comparing lattice calculations using different fermion formulations.
CKM 2008 LATTICE QCD

-1 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1
η
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
CKM 2008 LATTICE QCD
BBBf
sBB
sBf
KB
cbV
ubV
βγ
PDG06

* \( |V_{cb}| \) from RBC/UKQCD
* \( |V_{ub}| \) from Jack Laiho, LAT2007
* \( |V_{us}| \) from \( K_{exp} \) + HPQCD
* preliminary result from FNAL/MILC
* \( \frac{f_{B_s}}{f_{B}} \) from Flynn and Nieves, 0705.3553
* \( \hat{B}_k \) from RBC/UKQCD

C. Davies & C. McNeile
Other Heavy-light semileptonic decays

<table>
<thead>
<tr>
<th></th>
<th>Flavour neutral</th>
<th>Unstable</th>
<th>affordable now</th>
<th>in 5 years?</th>
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</thead>
<tbody>
<tr>
<td>$B \to \eta l\nu$</td>
<td>$\checkmark$</td>
<td></td>
<td>possible but</td>
<td></td>
</tr>
<tr>
<td>$B \to \eta' l\nu$</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
<td>expensive</td>
<td></td>
</tr>
<tr>
<td>$B \to \rho l\nu$</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
<td></td>
<td>$\checkmark$</td>
</tr>
<tr>
<td>$B \to \omega l\nu$</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
<td></td>
<td>$\checkmark$</td>
</tr>
<tr>
<td>$B \to K l\nu$</td>
<td></td>
<td>$\checkmark$</td>
<td></td>
<td>$\checkmark$</td>
</tr>
<tr>
<td>$B \to K^* l\nu$</td>
<td></td>
<td>$\checkmark$</td>
<td></td>
<td>$\checkmark$</td>
</tr>
<tr>
<td>$B \to \phi l\nu$</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
<td></td>
<td>$\checkmark$</td>
</tr>
<tr>
<td>$B \to K^* \gamma$</td>
<td></td>
<td>$\checkmark$</td>
<td></td>
<td>$\checkmark$</td>
</tr>
</tbody>
</table>

R. Van de Water
HISQ action

E. Follana et al, HPQCD coll.

- Highly improved staggered action.
- Much improved control of discretization errors.
  * Highly reduce $O(a^2 \alpha_s)$ errors (an order of magnitude)
  * Substantially reduce taste-changing with respect to Asqtad
  * No tree-level $O((am)^4)$ at first order in the quark velocity $v/c$
    → accurate results for charm quarks
## Error budget for decay constants

<table>
<thead>
<tr>
<th>Source</th>
<th>$f_\pi$</th>
<th>$f_K$</th>
<th>$f_K/f_\pi$</th>
<th>$f_D$</th>
<th>$f_{D_s}$</th>
<th>$f_{D_s}/f_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$ uncert.</td>
<td>1.4</td>
<td>1.1</td>
<td>0.3</td>
<td>1.4</td>
<td>1.0</td>
<td>0.4</td>
</tr>
<tr>
<td>$a^2$ extrap.</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>finite volume</td>
<td>0.8</td>
<td>0.4</td>
<td>0.4</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>$m_{u/s}$ extrap.</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>statistical</td>
<td>0.5</td>
<td>0.4</td>
<td>0.2</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
</tr>
<tr>
<td>$m_s$ evol.</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>$m_d$, QED, etc</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td><strong>Total(%)</strong></td>
<td>1.7</td>
<td>1.3</td>
<td>0.6</td>
<td>1.8</td>
<td>1.3</td>
<td>0.9</td>
</tr>
</tbody>
</table>
# $m_c$ extracted from current-current correlators.

* **HISQ** action used to determine moments $G_n$ of charm-quark pseudoscalar, vector and axial-vector correlators.

$$G_n \equiv \sum_t (t/a)^n G(t)$$

with

$$G(t) \equiv a^6 \sum_{\vec{x}} (am_{0c})^2 \langle 0 | J(\vec{x}, t) J(0, 0) | 0 \rangle$$

* Four-loop results from continuum perturbation theory for the moments.

$\begin{align*}
  m_c(m_c) &= 1.266(16) GeV \\
  m_c(3 GeV) &= 0.983(13) GeV
\end{align*}$
Same programme can be applied for extra operators

\[ \langle \bar{B}_0^0 d(s) | Q_{i=1-5} | B_0^0 d(s) \rangle \]

- Chiral perturbation theory more complicated (extra free parameters):

\[ \langle \bar{B}_0^0 d(s) | Q_{i=1-5} | B_0^0 d(s) \rangle \to_{chiral} \Gamma_i (1 + L) + \Gamma_i' L' + \text{analytic terms} \]
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\langle \overline{B}_0^{0(d(s)}} | Q_{i=1-5} | B_0^{0(d(s)}} \rangle
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\]

# Complete \( N_{f+1} \) analysis of \( \Delta B = 2 \) matrix elements expected from both Fermilab lattice-MILC and HPQCD collaborations in 1-2 years with errors < 10%.

* **First results**: One-loop renormalization for HPQCD study

( E.G, Shigemitsu, Trottier)