Hyperon Physics from Lattice QCD

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Outline

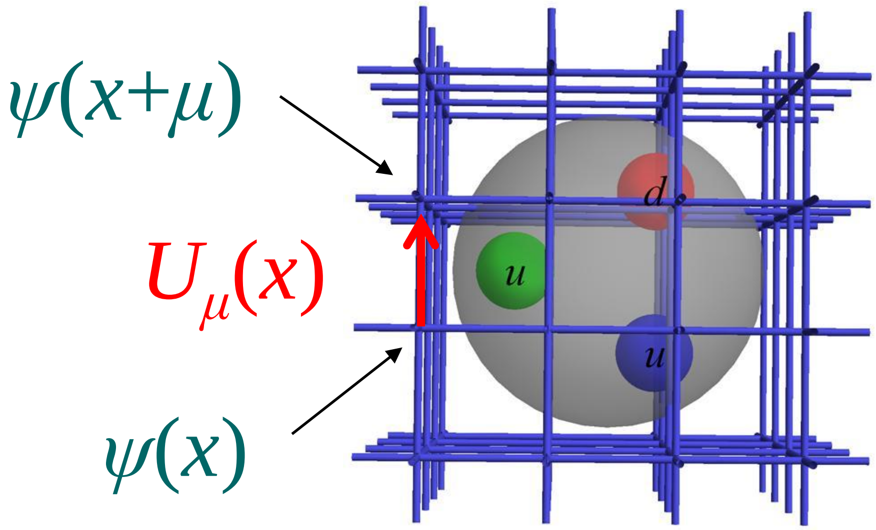
- Lattice QCD 101
- Hyperon spectroscopy
- Hyperon axial charge couplings
- Hyperon EM form factors
- Strangeness in the nucleon
- Hyperon semileptonic decays

Lattice QCD

- Physical observables are calculated from the path integral

$$\langle 0|O(\overline{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int [dA][d\overline{\psi}][d\psi]\, O(\overline{\psi},\psi,A)e^{i\int d^4x\,\mathcal{L}^{\rm QCD}(\overline{\psi},\psi,A)}$$

- Strong coupling regions: expansion no longer converge
- Lattice QCD is a discrete version of continuum QCD theory



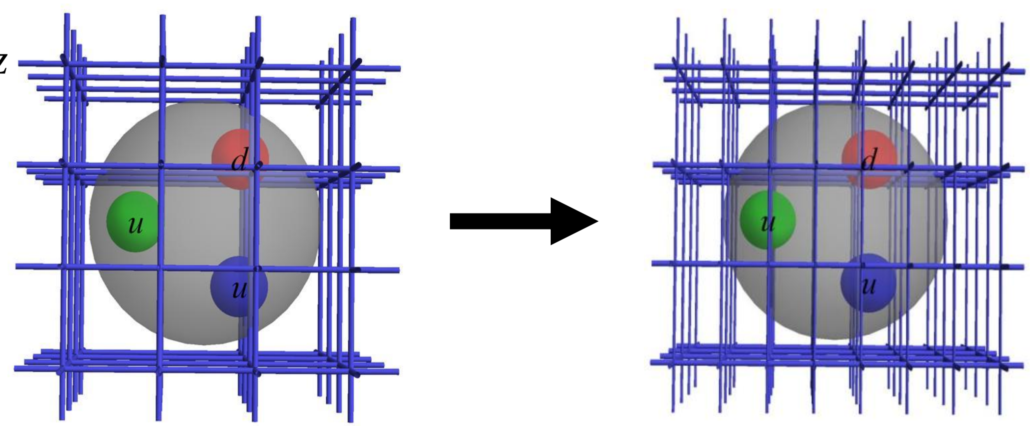
- Numerical integration to calculate the path integral
- Take $a \to 0$ and $V \to \infty$ in the continuum limit

Lattice Actions

- Symanzik Improvement
  - Order-by-order in a improvement of the action and operators
  - Systematic error due to discretization under control
- Gauge actions
  - Most gauge actions used today are $O(a^2)$ improved
  - Small discretization effects ($\sim O(\Lambda_{QCD}^3 a^3)$) due to gauge choices
- Fermion actions
  - Most fermion actions are only $O(a)$ improved ($O(\Lambda_{QCD}^2 a^2)$)
  - Differences are benign once all systematics are included
  - Different choices of fermion action are confined by limits of computational and human power + by personal interest
  - Commonly known actions: Domain-wall fermions, overlap fermions, Wilson/Clover fermions, twisted-Wilson fermions

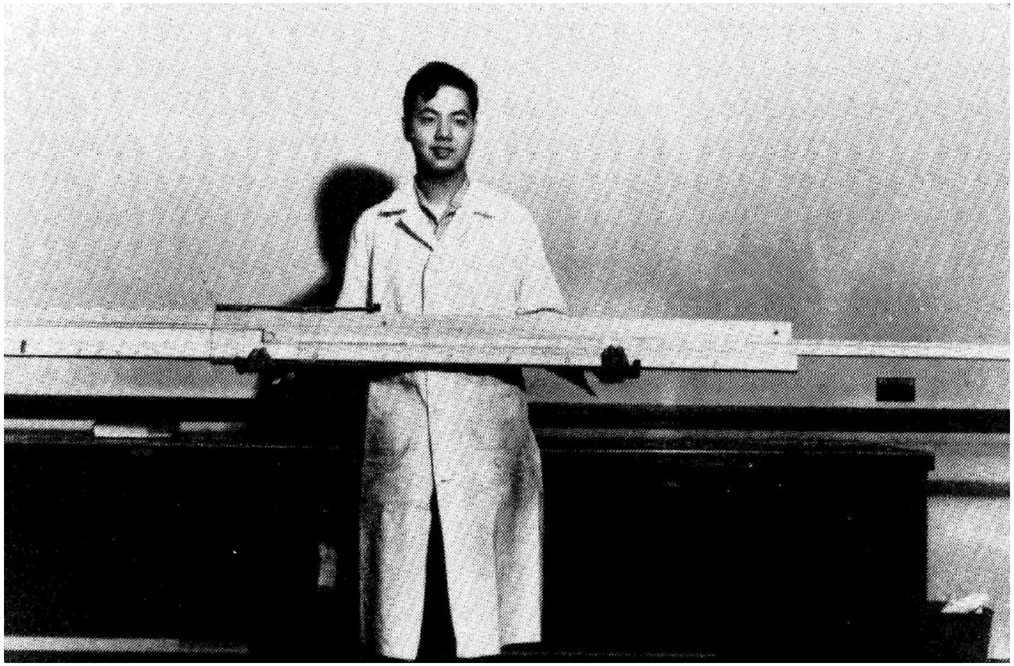
Not-So-Conventional Choices…

- Mixed Action
  - Example: Staggered sea with domain-wall valence
  - Staggered: Relatively cheap for dynamical fermions but nightmare for baryonic operators
  - DWF: chiral symmetry preserved on finite a; good for spin physics and weak matrix elements
- Anisotropic Wilson/Clover
  - Wilson/Clover fermions with broken space/time symmetry
  - Lattice spacing $a_t < a_{x,y,z}$
  - Complicated but useful for excited-state physics



Computational Requirements

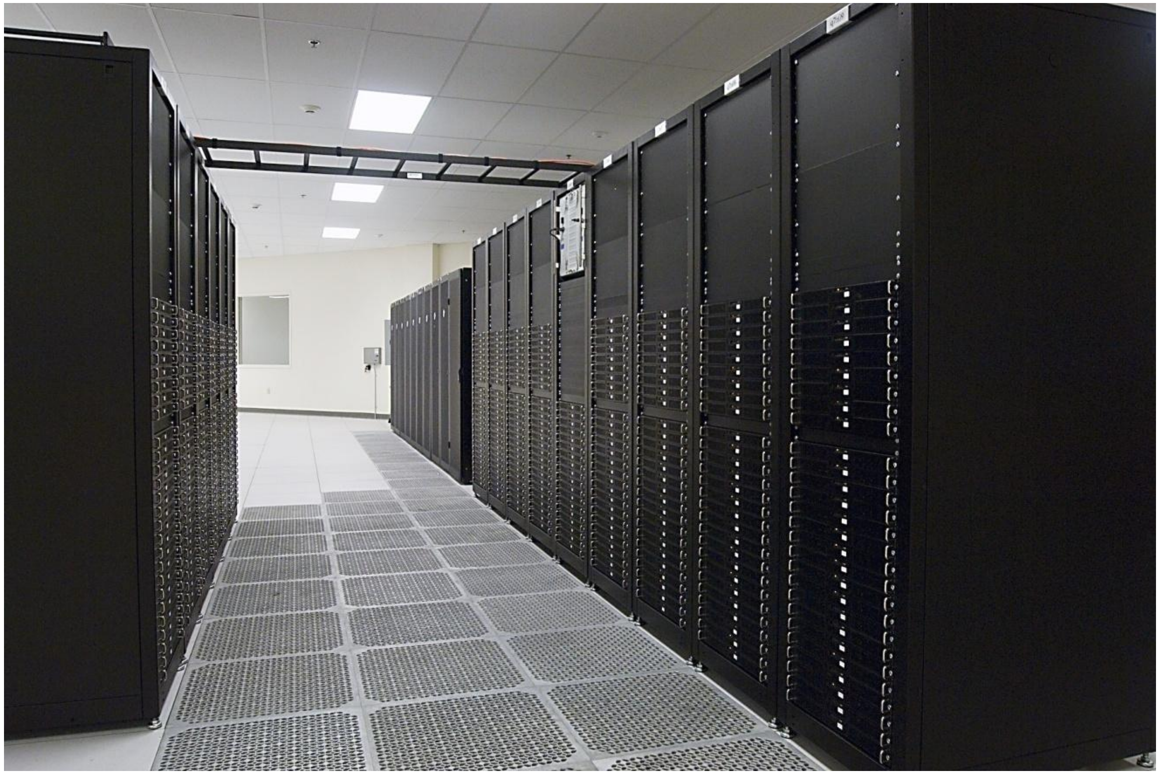
- In 1970, Wilson started off by writing down the first actions
- Why haven't we solved QCD yet?
  - Progress is limited by computational resources
  - But assisted by advances in algorithms
- Trace back to the my academic grandfather's generation



To calculate
stellar radiative transfer
equations, T.D. Lee uses
an "analog computer"

Computational Requirements

2007: The 13 Tflops cluster at Jefferson Lab



Other joint lattice resources within the US: Fermilab, BNL

Non-lattice resources open to USQCD: ORNL, LLNL, ANL

Computational Requirements

- Gauge generation costs with the latest algorithms scale like

  Cost factor: a^{-6}, L^5, M_π^{-3}

- Most of the major 2+1-flavor gauge ensembles:

  $M_\pi < 300$ MeV

- Chiral domain-wall fermions (DWF)

  at large volume (6 fm)
  at physical pion mass may be
  expected in 2011

Norman Christ, LAT2007

- PACS-CS (Clover action) report preliminary results at
  $M_\pi \sim 150$ MeV (but small volume) at Lat@ECT (2008 May)

- But for now….
  need a pion mass extrapolation $M_\pi \rightarrow (M_\pi)_{\text{phys}}$
  (use chiral perturbation theory, if available)

Systematic Errors

- Currently, not at the physical pion-mass point
  XPT uncertainty (parameters used in XPT, etc.)

- Finite lattice spacing
  - Exact: Do multiple lattice-spacing calculations and extrapolate to $a = 0$
  - Otherwise, estimate according to the level of improvement for the gluon and fermion action and operators

- Finite-volume effect
  - Exact: Do multiple volume calculations and extrapolate to $V = \infty$
  - Otherwise, estimate according to previous work
  - Or apply finite-volume XPT to try to correct FVE

- Other systematics
  - For example: if fitting is involved, what is the dependence on the fit range?

Hyperon Resonances

in collaboration with

David Richards, and other members in LHPC

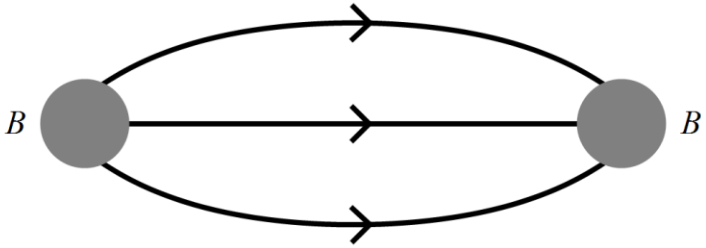
Spectroscopy on Lattice

- Calculate two-point Green function

$$\begin{aligned}\langle O\rangle &= \frac{1}{Z}\int [dU][d\psi][d\overline{\psi}]e^{-S_F(U,\psi,\overline{\psi})-S_G(U)}O(U,\psi,\overline{\psi}) \\ &= \frac{1}{Z}\int [dU]\det M e^{-S_G(U)}O(U)\end{aligned}$$

- Spin projection

$$\sum_{\alpha,\beta}\Gamma^{\alpha,\beta}\langle J(X_{\text{snk}})J(X_{\text{src}})\rangle_{\alpha,\beta}$$



- Momentum projection

Two-point correlator

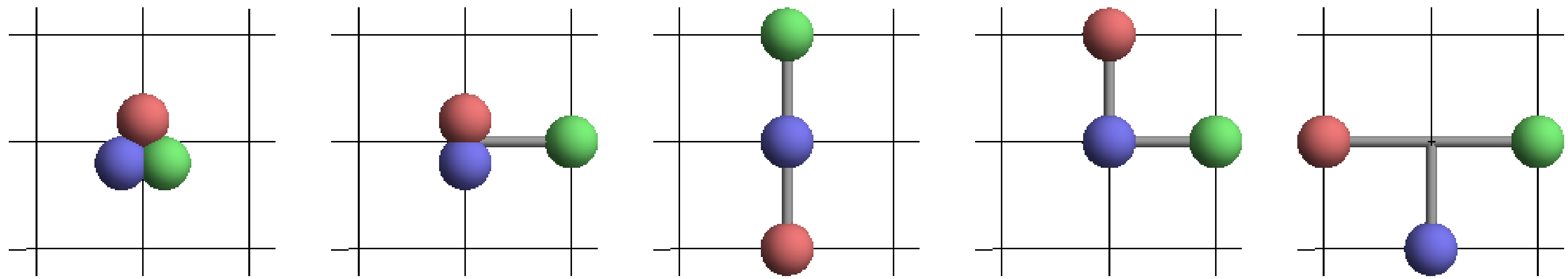
Exp decay

$$\Gamma^{(2),T}_{AB}(t;\vec{p}) = \sum_n \frac{E_n+M_n}{2E_n} Z_{n,A} Z_{n,B} e^{-E_n(\vec{P})t}$$

At large enough *t*, the ground-state signal dominates

Operator Design

- Baryon field $\Phi^{ABC}_{\alpha\beta\gamma,ijk}(x) = \epsilon_{abc}[\tilde{D}_i^{(3)}\tilde{\psi}]_{Aa\alpha}(x)[\tilde{D}_j^{(3)}\tilde{\psi}]_{Bb\beta}(x)[\tilde{D}_k^{(3)}\tilde{\psi}]_{Cc\gamma}(x)$



- Classify states according to symmetry properties
- Projection onto irreducible representations of finite groups
- Number of operators:

N^+ Operator type	G_{1g}	H_g	G_{2g}
Single-Site	3	1	0
Singly-Displaced	24	32	8
Doubly-Displaced-I	24	32	8
Doubly-Displaced-L	64	128	64
Triply-Displaced-T	64	128	64
Total	**179**	**321**	**144**

S. Basak et al., Phys. Rev. D72, 094506 (2005)

Variational Method

- Construct the correlator matrix

$$C_\Lambda^{m,n}(t) = \sum_{\vec{x}} \sum_{\lambda} \langle 0 \mid B_\lambda^{\Lambda,m}(\vec{x},t) \bar{B}_\lambda^{\Lambda,n}(0) \mid 0 \rangle$$

- Construct the matrix

$$C_{i\ j}(t) = \langle 0 \mid \mathcal{O}_i(t)^\dagger \mathcal{O}_j(0) \mid 0 \rangle$$

- Solve for the generalized eigensystem of

$$C(t)\psi = \lambda(t,t_0) C(t_0)\psi$$

with eigenvalues

$$\lambda_n(t,t_0) = e^{-(t-t_0)E_n}\left(1 + \mathcal{O}(e^{-|\delta E|(t-t_0)})\right)$$

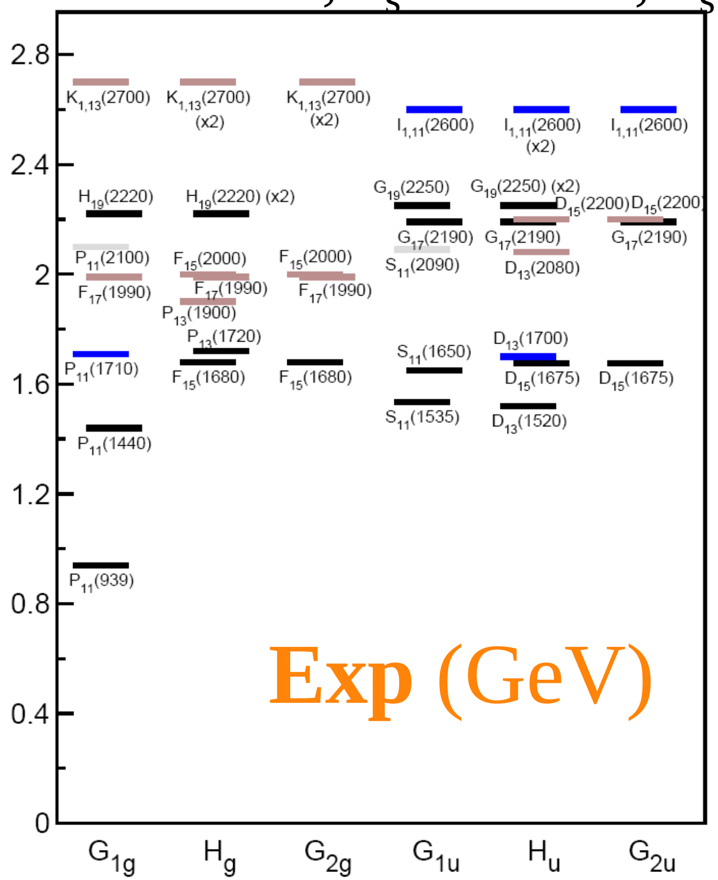
C. Michael, Nucl. Phys. B 259, 58 (1985)

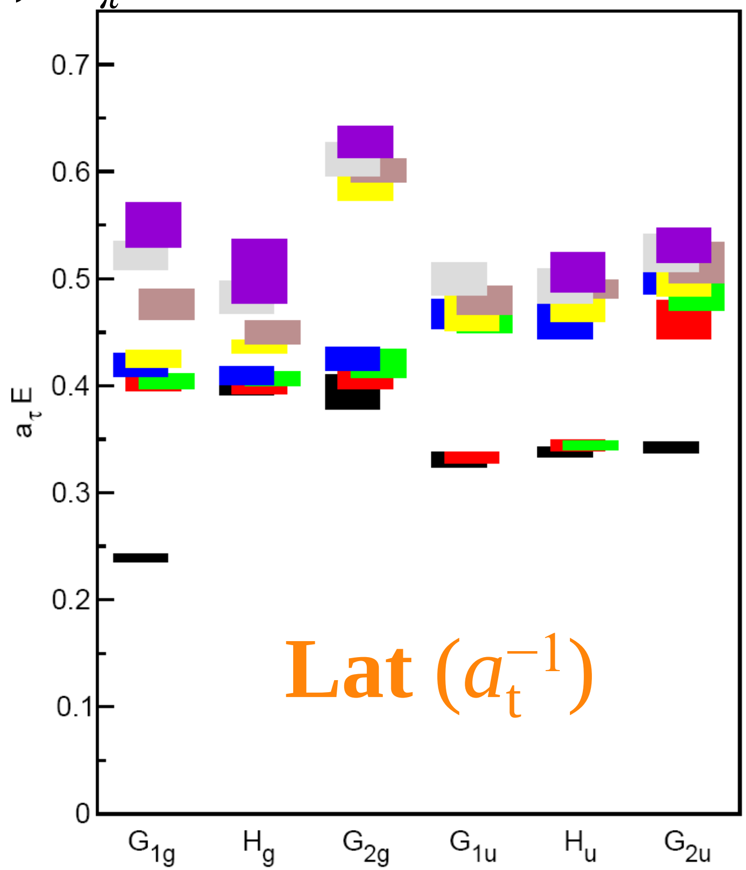
M. Lüscher and U. Wolff, Nucl. Phys. B 339, 222 (1990)

- At large *t*, the signal of the desired state dominates.

Does It Work?

- Nucleon on anisotropic Wilson action (quenched), hep-lat/0609019
  $V = 12^3 \times 48$, $a_s \sim 0.1$ fm, $a_s/a_t \sim 3$, $M_\pi \sim 700$ MeV





- $N_f = 2$ light and strange baryons are on the way
- $N_f = 2+1$ gauge generation is almost complete

Full QCD?

- Examples of a $N_f = 2+1$ study
  - *Isotropic* mixed action: DWF on staggered sea
  - $M_\pi \sim 300–750$ MeV, $L \sim 2.5$ fm
  - Number of operators:

Flavor	$G_{1g/u}(2)$	$H_{g/u}(4)$
N	3	1
Δ	1	2
Λ	4	1
Σ	4	3
Ξ	4	3
Ω	1	2

  - Naïve chiral extrapolation

    XPT for most of the excited states is unknown

j	Irreps
$\frac{1}{2}$	G_1
$\frac{3}{2}$	H
$\frac{5}{2}$	$G_2 \oplus H$
$\frac{21}{2}$	$G_1 \oplus 2\,G_2 \oplus 4\,H$
$\frac{23}{2}$	$2\,G_1 \oplus 2\,G_2 \oplus 4\,H$

This calculation:

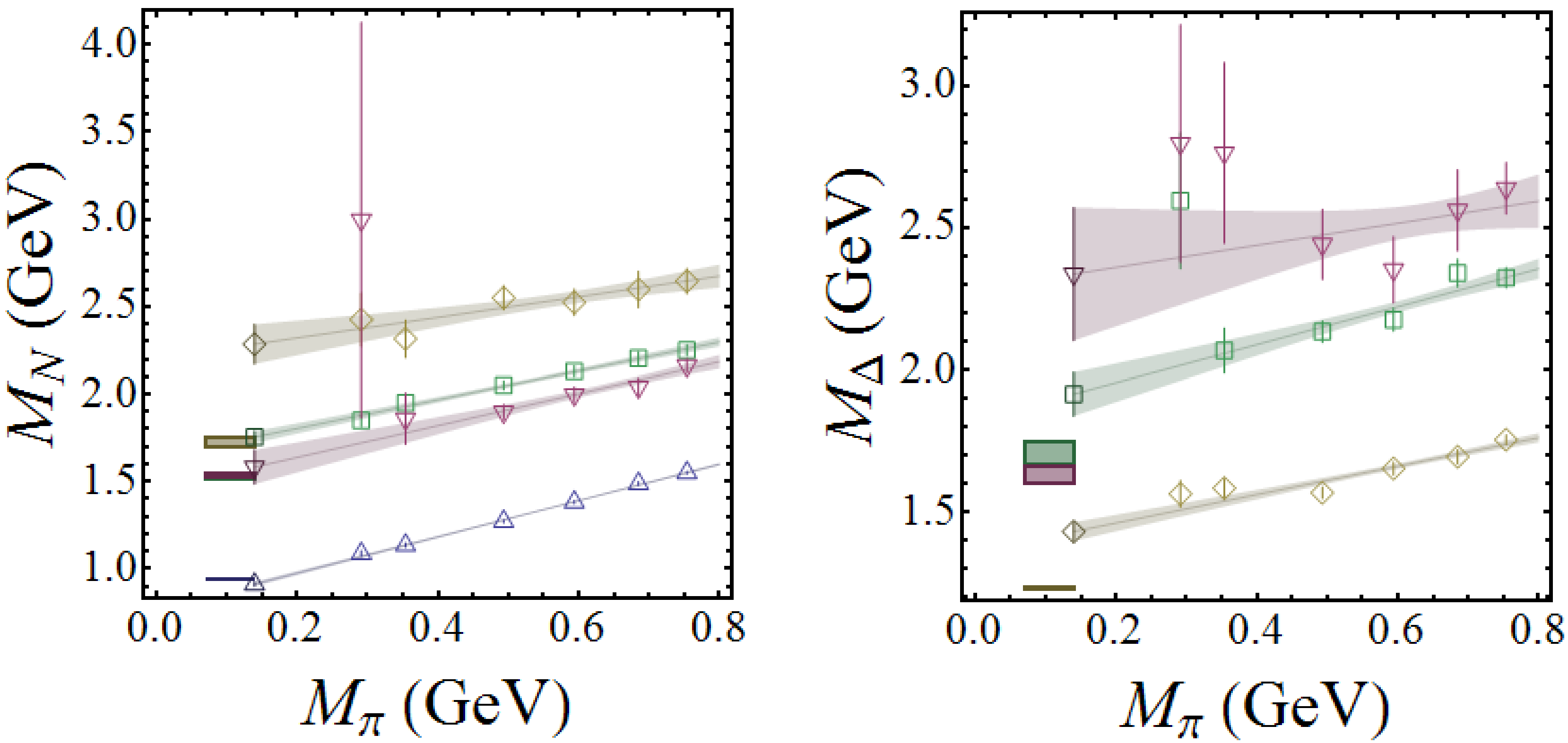
Three quarks in a baryon located at a single site

Non-Strange Baryons

2+1-flavor mixed action

- The non-strange baryons (*N* and Δ)
- Symbols: J^P = 1/2⁺ △, 1/2⁻ ▽ , 3/2⁺ ◇, 3/2⁻ □

1/2⁺ △	1/2⁻ ▽	3/2⁺ ◇	3/2⁻ □
N	*N*(1535)	*N*(1720)	*N*(1520)
	Δ(1620)	Δ	Δ(1700)

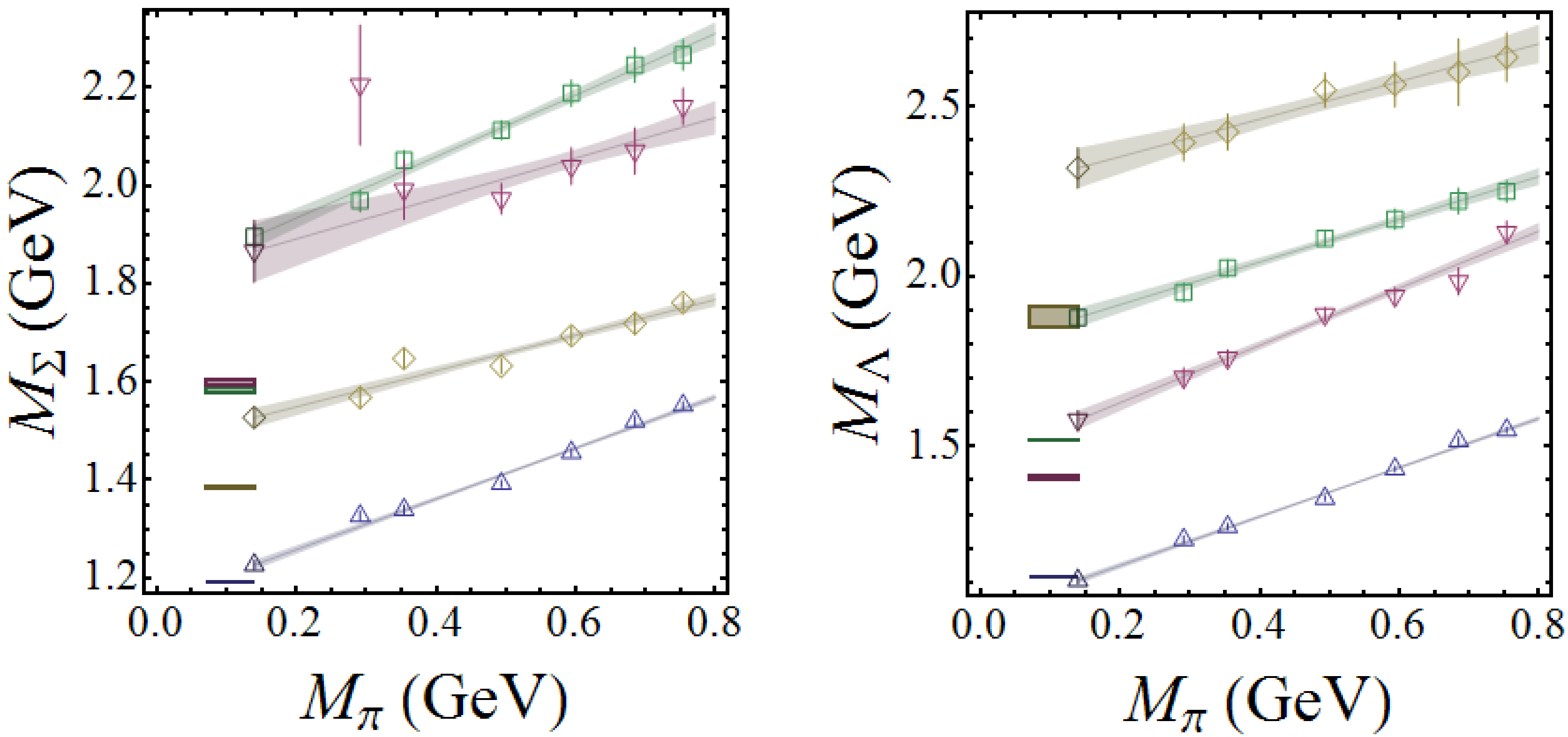


Singly Strange Baryons

2+1-flavor mixed action

- The singly strange baryons: (Σ and Λ)
- Symbols: J^P = 1/2$^+$ △, 1/2$^-$ ▽, 3/2$^+$ ◇, 3/2$^-$ □

1/2$^+$	1/2$^-$	3/2$^+$	3/2$^-$
Σ	Σ(1620)	Σ*	Σ(1580)
Λ	Λ(1405)	Λ(1890)	Λ(1520)

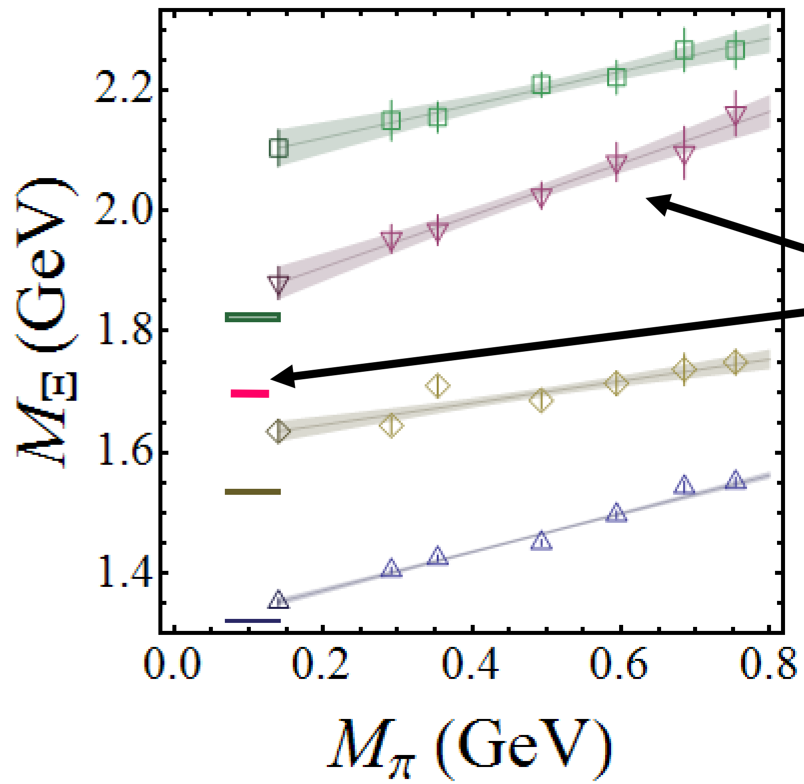


Really Strange Baryons

2+1-flavor mixed action

- The less known baryons (Ξ)
- Symbols: J^P = 1/2⁺ △, 1/2⁻ ▽ , 3/2⁺ ◇, 3/2⁻ □

Ξ　Ξ(1690)?　Ξ(1530)　Ξ(1820)



- BaBar at MENU 2007: Ξ(1690)⁰ negative parity −1/2

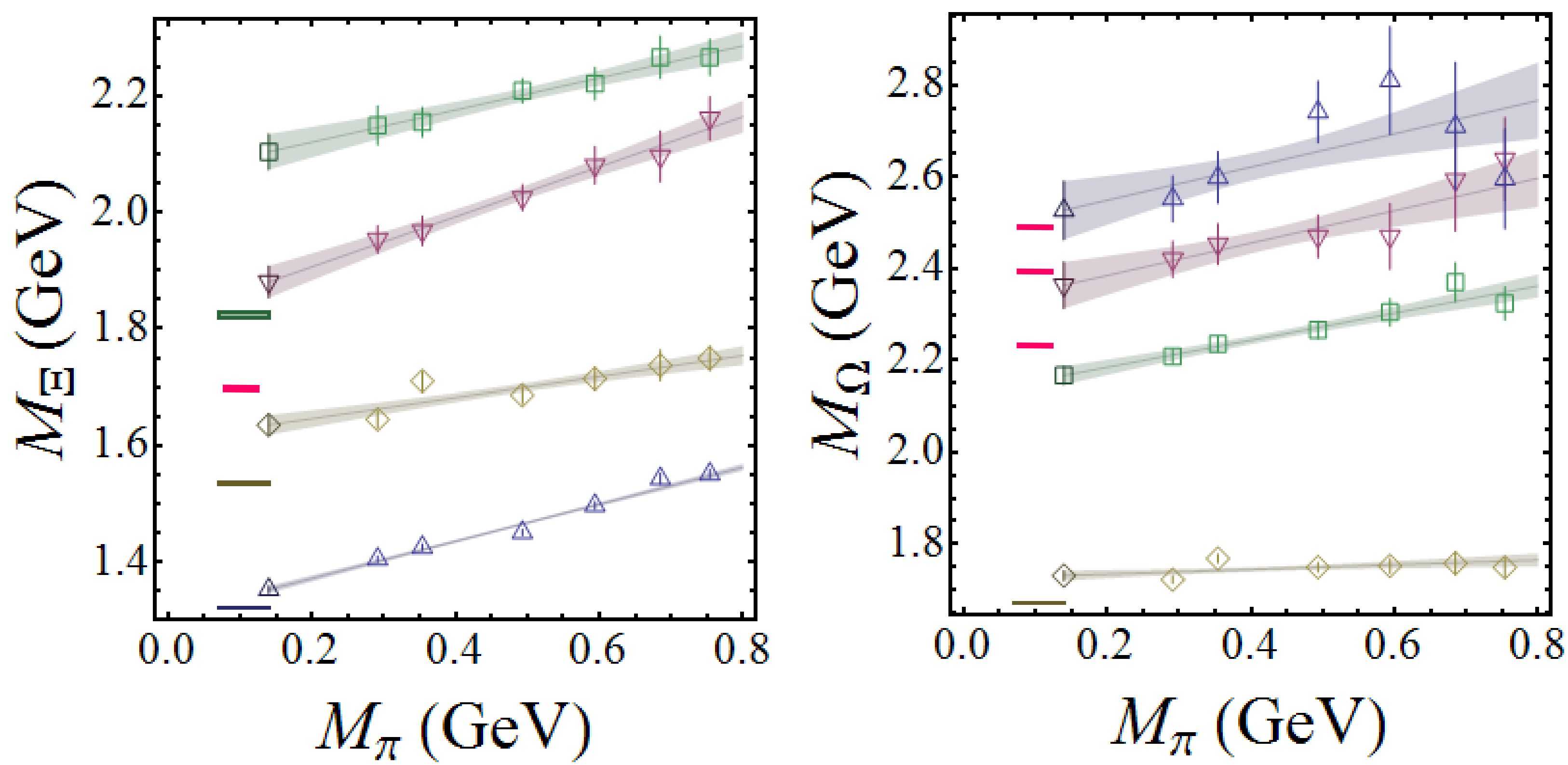
Really Strange Baryons

2+1-flavor mixed action

- The less known baryons (Ξ and Ω)
- Symbols: J^P = 1/2⁺ △ , 1/2⁻ ▽ , 3/2⁺ ◇, 3/2⁻ □

  Ξ, Ξ(1690)?, Ξ(1530), Ξ(1820)

Spin prediction for Ω(2250), Ω(2380), Ω(2470)?



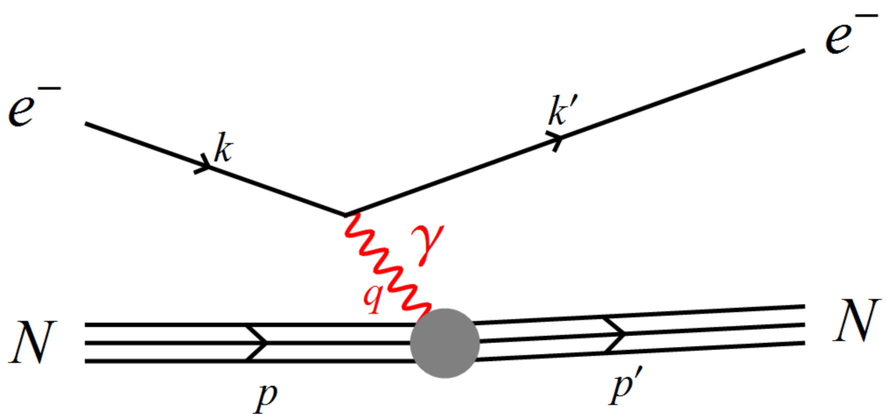
Axial Coupling Constants

in collaboration with

Kostas Orginos

Axial Couplings and Form Factors

- Elastic scattering process
  - Axial couplings
  - Vector and axial form factors
  - Magnetic moments
  - Charge radii



- For octet baryons

$$\langle B\,|V_\mu|\,B\rangle(q) \;=\; \overline{u}_B(p')\left[\gamma_\mu F_1(q^2)+\sigma_{\mu\nu}q_\nu\frac{F_2(q^2)}{2M_B}\right]u_B(p)$$

$$\langle B\,|A_\mu(q)|\,B\rangle \;=\; \overline{u}_B(p')\left[\gamma_\mu\gamma_5 G_A(q^2)+\gamma_5 q_\nu\frac{G_P(q^2)}{2M_B}\right]u_B(p)$$

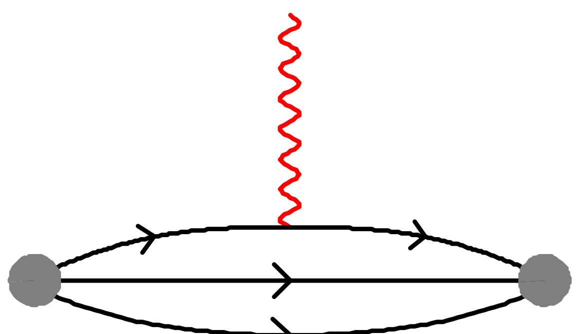
Green Functions

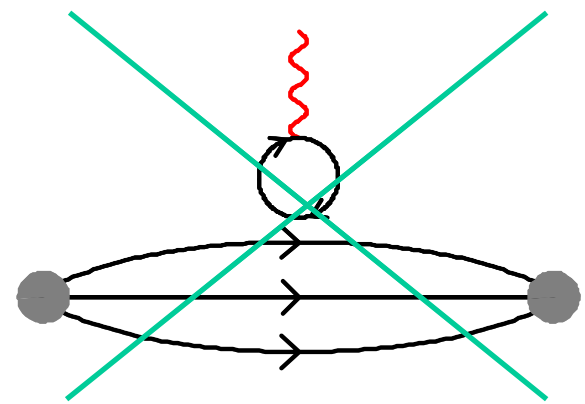
- Three-point function with connected piece only

$$C_{3\mathrm{pt}}^{\Gamma,\mathcal{O}}\left(\vec{p},t,\tau\right)=\sum_{\alpha,\beta}\Gamma^{\alpha,\beta}\langle J_\beta\left(\vec{p},t\right)\mathcal{O}(\tau)\overline{J}_\alpha\left(\vec{p},0\right)\rangle$$

$$\mathrm{O} : \mathrm{V}_\mu = \bar{q}\gamma_\mu q,\ \ \mathrm{A}_\mu = \bar{q}\gamma_\mu\gamma_5 q,\ J = \epsilon^{abc}[q_1^{a\mathrm{T}}(x)C\gamma_5 q_2^b(x)]q_1^c(x)$$

- Two topologies:





- Isovector quantities O^{u-d}: disconnected diagram cancelled

Axial Coupling Constants: $g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

- Define $\langle B|A_\mu(q)|B\rangle = \overline{u}_B(p')\left[\gamma_\mu\gamma_5 G_A(q^2) + \gamma_5 q_\nu \frac{G_P(q^2)}{2M_B}\right]u_B(p)e^{-iq\cdot x}$
- Has applications such as hyperon scattering, non-leptonic decays, …
- Cannot be determined by experiment
- Existing theoretical predictions:
  - Chiral perturbation theory

$$0.35 \le g_{\Sigma\Sigma} \le 0.55 \qquad 0.18 \le -g_{\Xi\Xi} \le 0.36$$

M. J. Savage et al., Phys. Rev. D55, 5376 (1997);

  - Large-N_c

$$0.30 \le g_{\Sigma\Sigma} \le 0.36 \qquad 0.26 \le -g_{\Xi\Xi} \le 0.30$$

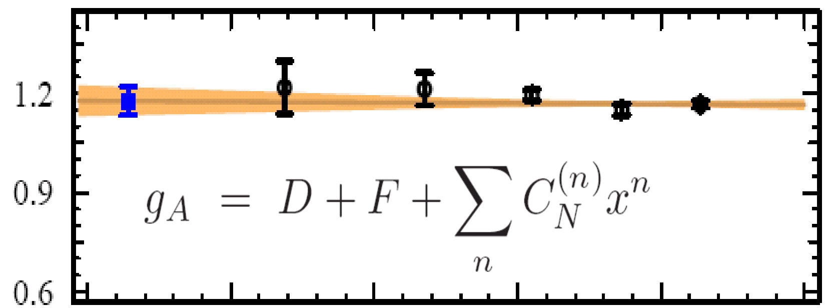
R. Flores-Mendieta et al., Phys. Rev. D58, 094028 (1998);

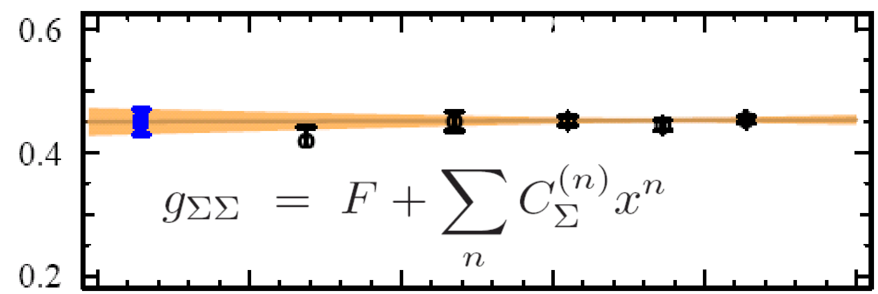
- Loose bounds on the values
- Lattice QCD can provide substantial improvement
  - Pion mass: 350–750 MeV on mixed action

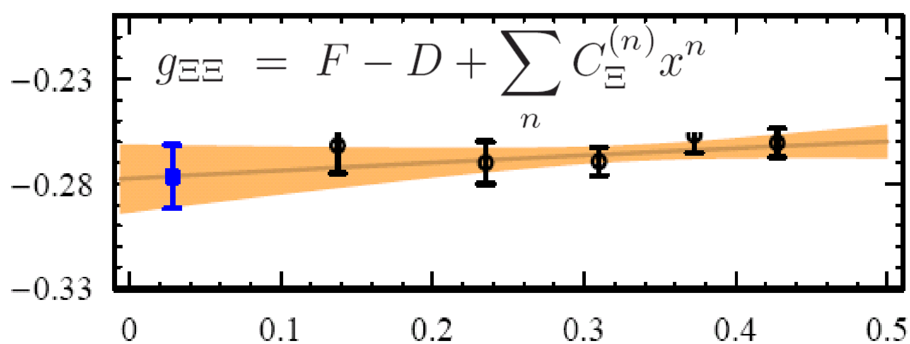
HWL and K. Orginos, arXiv:0712.1214

Axial Coupling Constants: $g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

- Simultaneous SU(3) fit
  - SU(3) chiral perturbation theory (with 8 parameters) which fails to describe the data
  - Simplified parametrization chiral form using $x = (m_K^2 - m_\pi^2)/(4\pi f_\pi^2)$







- Systematic errors: finite volume + finite *a*

$$g_A = 1.18(4)_{\rm stat}(6)_{\rm syst}$$
$$g_{\Sigma\Sigma} = 0.450(21)_{\rm stat}(27)_{\rm syst}$$
$$g_{\Xi\Xi} = -0.277(15)_{\rm stat}(19)_{\rm syst}$$

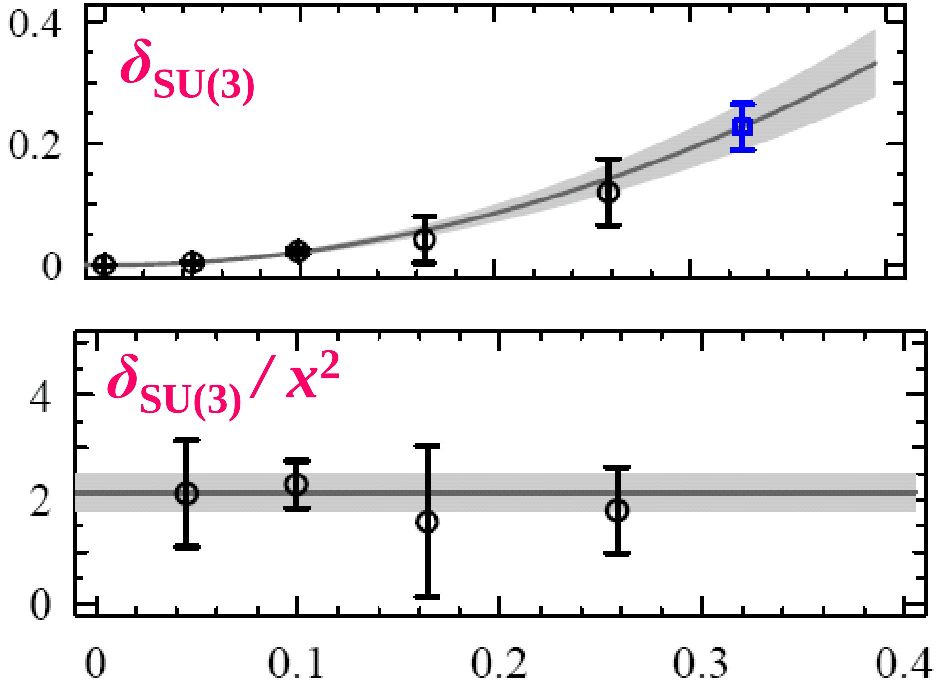
- Global coupling constants: $D = 0.715(6)(29)$ & $F = 0.453(5)(19)$

Axial Coupling Constants: $g_{\Xi\Xi}$ and $g_{\Sigma\Sigma}$

- SU(3) symmetry breaking

$$\delta_{\mathrm{SU(3)}} = g_{\mathrm{A}} - 2.0 \times g_{\Sigma\Sigma} + g_{\Xi\Xi}$$
$$= \sum_n c_n x^n \quad \text{with} \quad x = (m_K^2 - m_\pi^2)/(4\pi f_\pi^2)$$

- Quadratic behaviour is observed



- Not predicted by any theorem nor chiral perturbation theory $\Longrightarrow$ coincidence?
- 20% breaking at physical point

Hyperon Form Factors

in collaboration with

Kostas Orginos

Electromagnetic Form Factors

- Two definitions
  - Dirac and Pauli form factors F_1, F_2

$$\langle N | V_\mu | N \rangle(q) = \overline{u}_N(p') \left[\gamma_\mu F_1(q^2) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{2m} \right] u_N(p)$$

At $Q^2 = 0$,

$$F_{1p}(0) = 1,\, F_{2p}(0) = \kappa_p,\, F_{1n}(0) = 0,\, F_{2n}(0) = \kappa_n$$

  - Sachs form factors G_E, G_M

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{(2M_N)^2} F_2(q^2)$$
$$G_M(q^2) = F_1(q^2) + F_2(q^2) \quad .$$

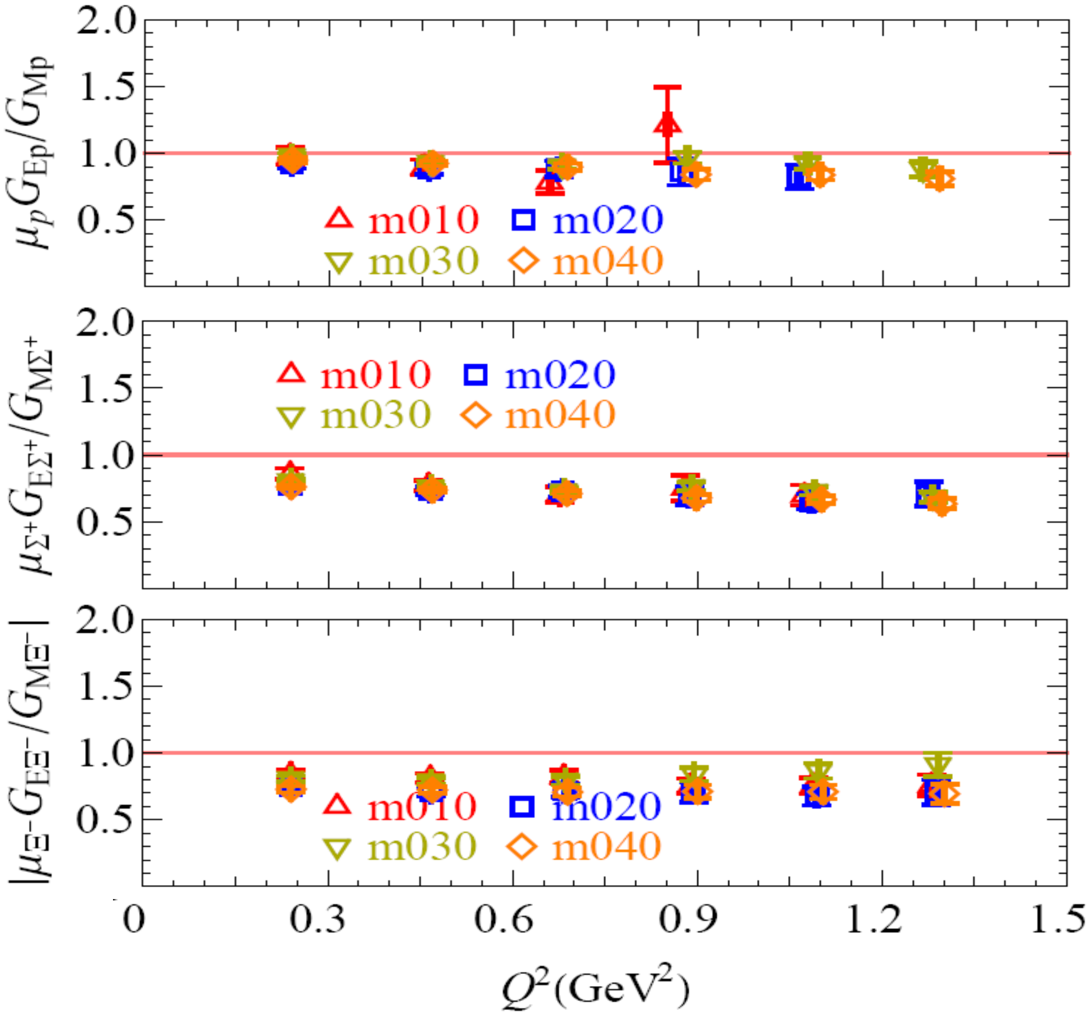
At $Q^2 = 0$,

$$G_{Ep}(0) = 1,\, G_{Mp}(0) = \mu_p,\, G_{En}(0) = 0,\, G_{Mn}(0) = \mu_n$$

- Isovector quantities only

Q^2-Dependence of Form Factors

- Ratio of $\mu_B G_{E,B}/G_{M,B}$ for p, Σ^+ and Ξ^-



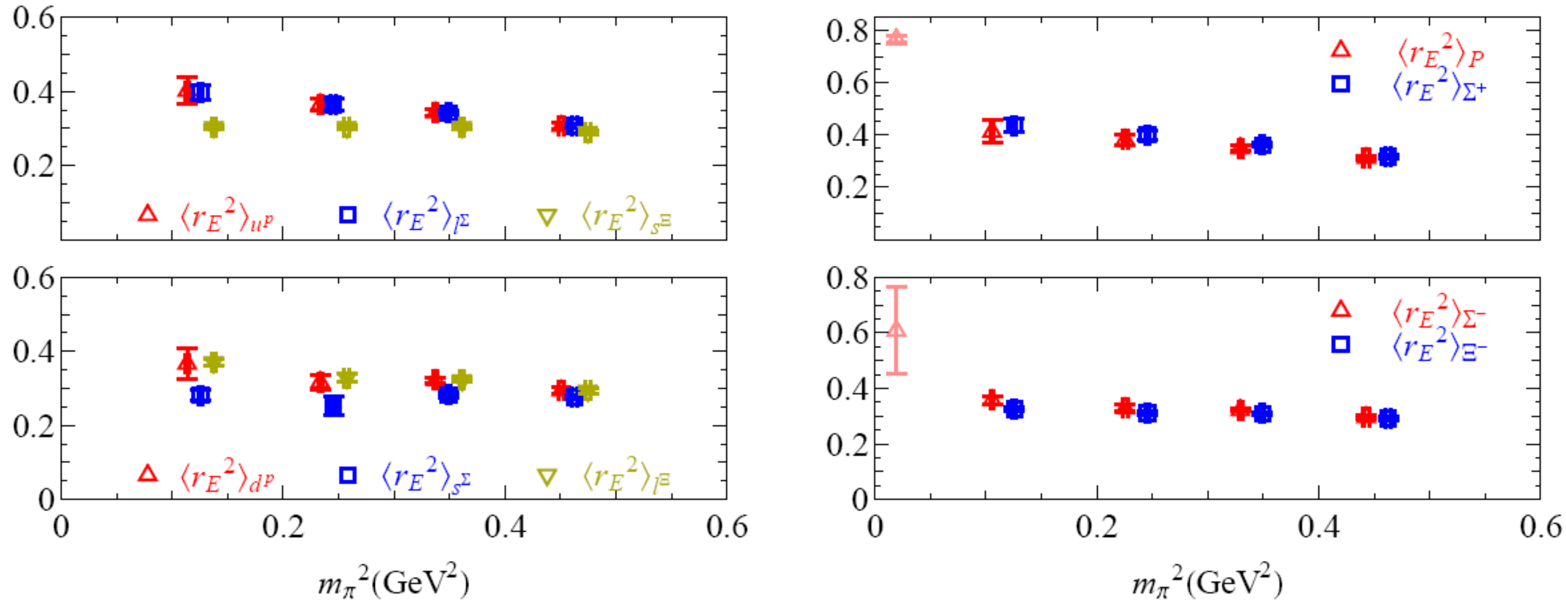
- Almost constant versus Q^2
- Symmetric

$$\langle r_M^2 \rangle \approx \langle r_E^2 \rangle$$

Charge Radii

- Electric charge radii $\langle r_E^2 \rangle = (-6)\frac{d}{dQ^2}\left(\frac{G_E(Q^2)}{G_E(0)}\right)|_{Q^2=0}$

Quark Level | Baryon Level

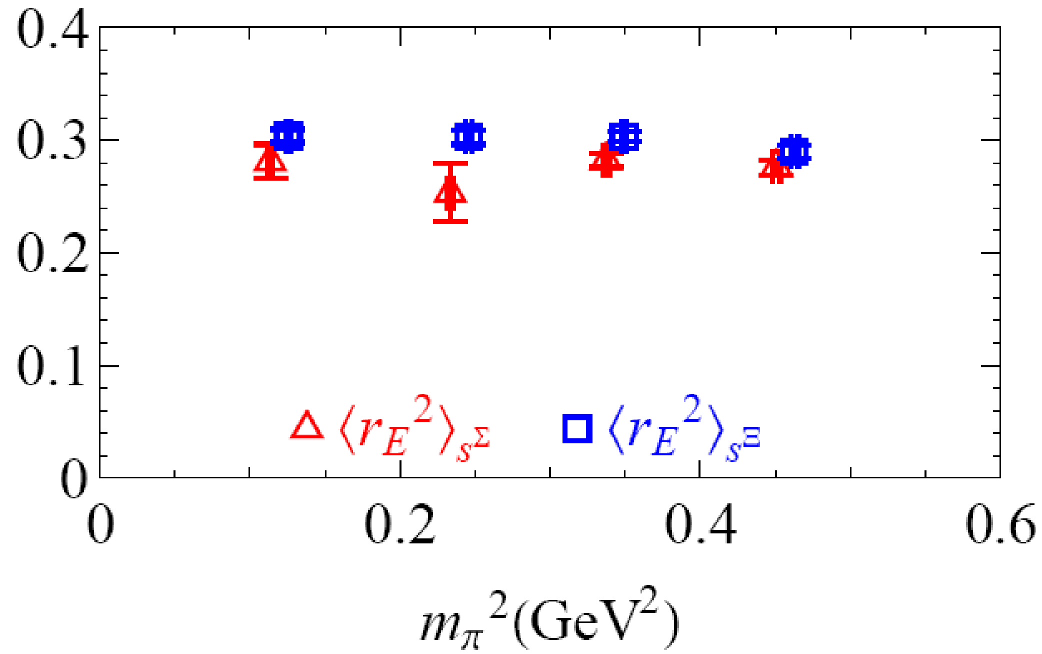


- Smaller strange contribution to charge radii ← shorter Compton wavelength
- u/d quark contribution seems to be independence of the environmental baryon
- Could provide predictions for Σ^+ and Ξ^-

Charge Radii

- Electric charge radii $\langle r_E^2 \rangle = (-6)\frac{d}{dQ^2}\left(\frac{G_E(Q^2)}{G_E(0)}\right)|_{Q^2=0}$

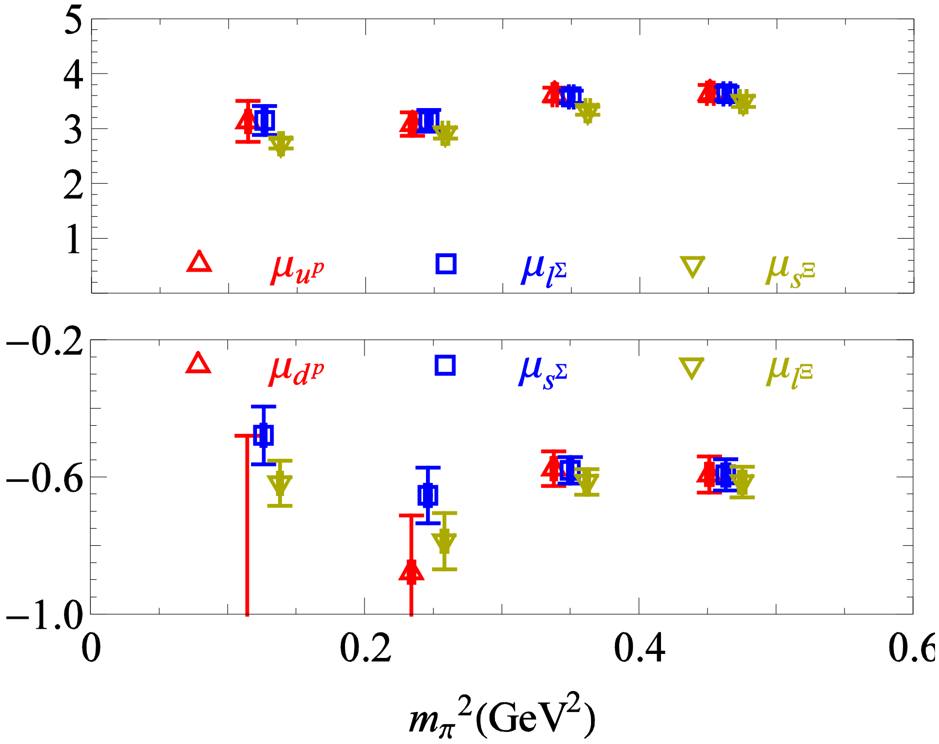
- Comparison between strange quark contributions: insensitive to environmental baryon



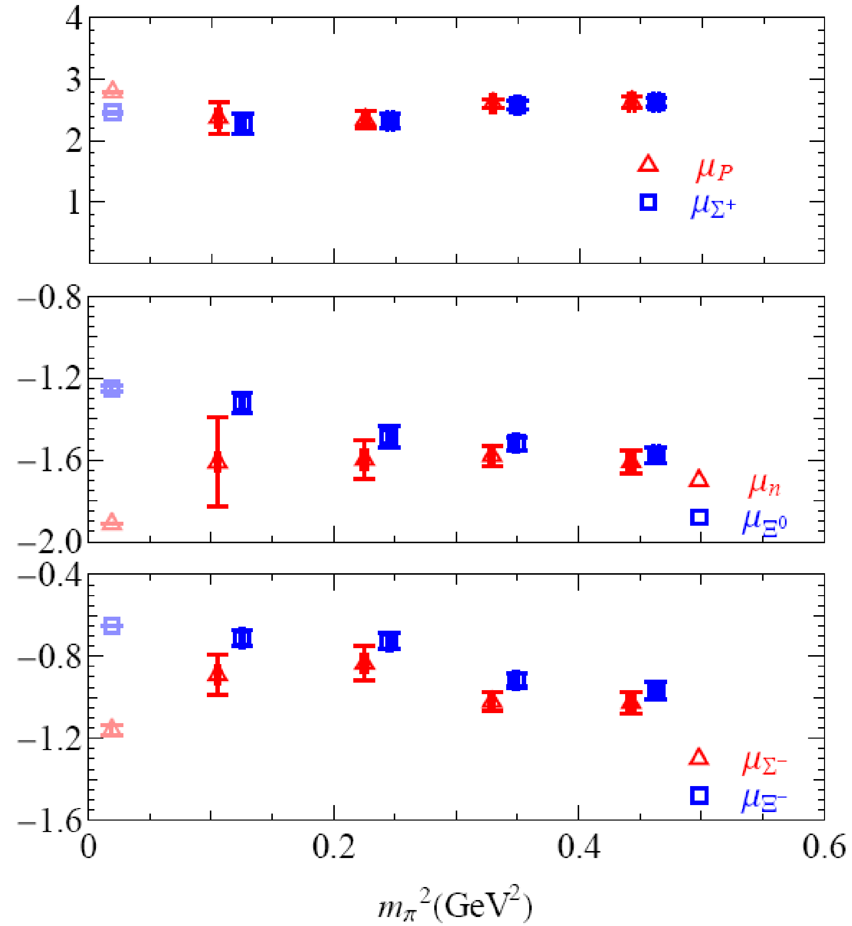
Magnetic Moments

- Magnetic moment $\mu_B = G_{M,B}(0) \times M_N/M_B$ dipole extrap. $G_{M,B}$

Quark Contribution



Baryon Contribution



Strange Magnetic Moment of Nucleon

- Purely sea-quark effect
- First strange magnetic moment was measured by SAMPLE

$$G_M^s(Q^2 = 0.1\ GeV^2) = 0.23(37)(25)(29)$$

B. Mueller et al. (SAMPLE) Phys. Rev. Lett. 78, 3824 (1997)

- New data, still being collected, suggests the value is non-zero.

HAPPEX and G0 collaborations at Jefferson Lab, SAMPLE at MIT-BATES, and A4 at Mainz

- Lattice calculations

$$\langle B\,|V_\mu|\,B\rangle(q) = \overline{u}_B(p')\left[\gamma_\mu F_1(q^2) + \sigma_{\mu\nu}q_\nu \frac{F_2(q^2)}{2M_B}\right]u_B(p)$$

The disconnected diagram is a must.

- Done in quenched approximation
  - Direct: Noisy ($\mathbf{Z}_2$) estimator — Kentucky Field Theory group (1997–2001)

    −0.28(10) to +0.05(6)
  - Indirect: Charge symmetry — Adelaide-JLab group (2006)

    −0.046(19)

Strange Magnetic Moment of Nucleon

- Two methods used in LQCD
  - Direct: all-to-all approach or noise estimators
  - Indirect: charge symmetry assumption (for example, $d^n = u^p$):

- Charge symmetry and Strangeness

D. B. Leinweber, Phys. Rev. D 53, 5115 (1996).

- The strangeness contribution in nucleon is

$$G_M^s = \left(\frac{{}^l R_d^s}{1 - {}^l R_d^s}\right)\left[2p + n - \frac{u^p}{u^\Sigma}(\Sigma^+ - \Sigma^-)\right]$$

$$G_M^s = \left(\frac{{}^l R_d^s}{1 - {}^l R_d^s}\right)\left[p + 2n - \frac{u^n}{u^\Xi}(\Xi^0 - \Xi^-)\right] \quad \text{with} \quad {}^l R_d^s \equiv {}^l G_M^s / {}^l G_M^d$$

Taken from Exp

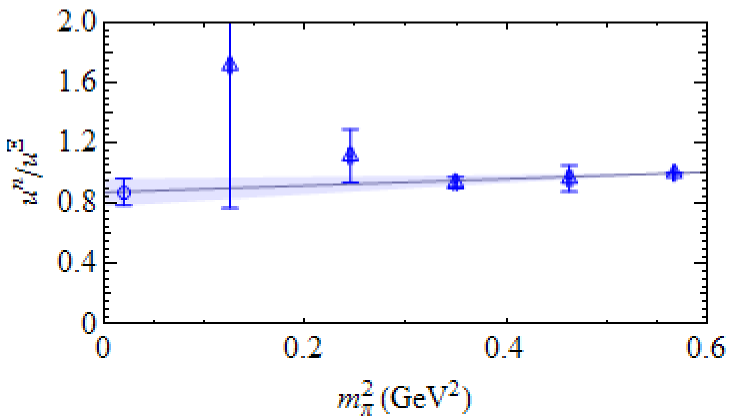
Cal. in LQCD

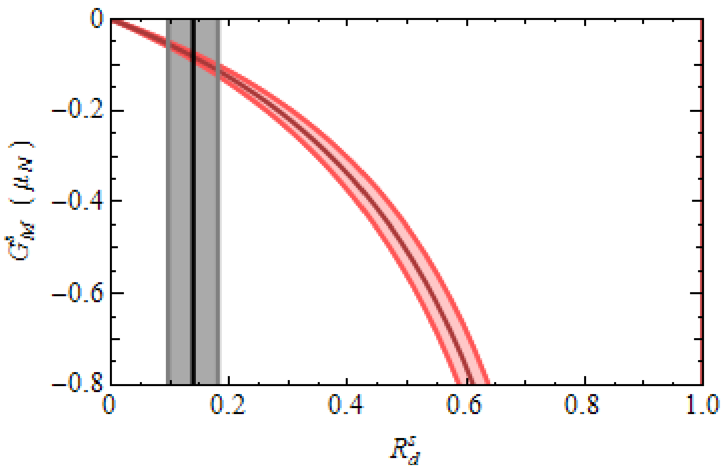
Strange Magnetic Moment of Nucleon

- Dipole-form extrapolation to $q^2 = 0$ to obtain μ_B
- Inputs from p, n for Σ^+ and Ξ^-

$$G_M^s = \left(\frac{{}^l R_d^s}{1 - {}^l R_d^s}\right)\left[-1.033 - \frac{u^n}{u^\Xi}(-0.599)\right]\mu_N$$

- Magnetic-moment ratios (linear extrapolation, for now)





- R_d^s estimated from XPT $\quad R_d^s = 0.139(42)$

D. B. Leinweber et al.,Phys. Rev. Lett. 94, 212001 (2004).

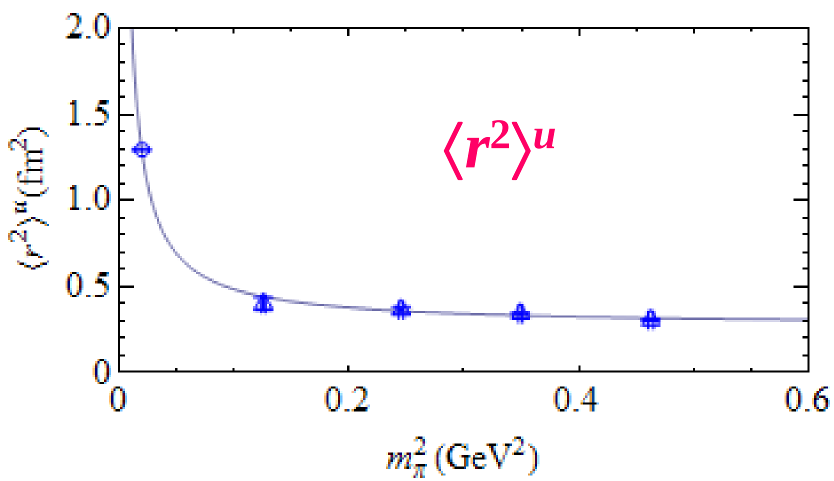
- We find $G_M^s = -0.082(8)_{\text{stat}}(25)_{\text{R}}$

Strange Electric Moment of Nucleon

- G_E^s is proportional to $Q^2\langle r^2\rangle^s$
- Charge symmetry: D. B. Leinweber et al., Phys. Rev. Lett. 94, 212001 (2004).

$$\langle r^2\rangle^s = \frac{r_d^s}{1-r_d^s}\left[2\langle r^2\rangle^p + \langle r^2\rangle^n - \langle r^2\rangle^u\right] \qquad r_d^s = 0.16(4)$$

- u-quark form contribution of vector form factors



- Need more literature research on chiral extrapolation
- Using an extrapolation of the form

$$\langle r^2\rangle^u = a_0 - \frac{1+5g_A^2}{(4\pi f_\pi)^2}\log\left(\frac{m_\pi^2}{m_\pi^2+\Lambda^2}\right)$$

We find

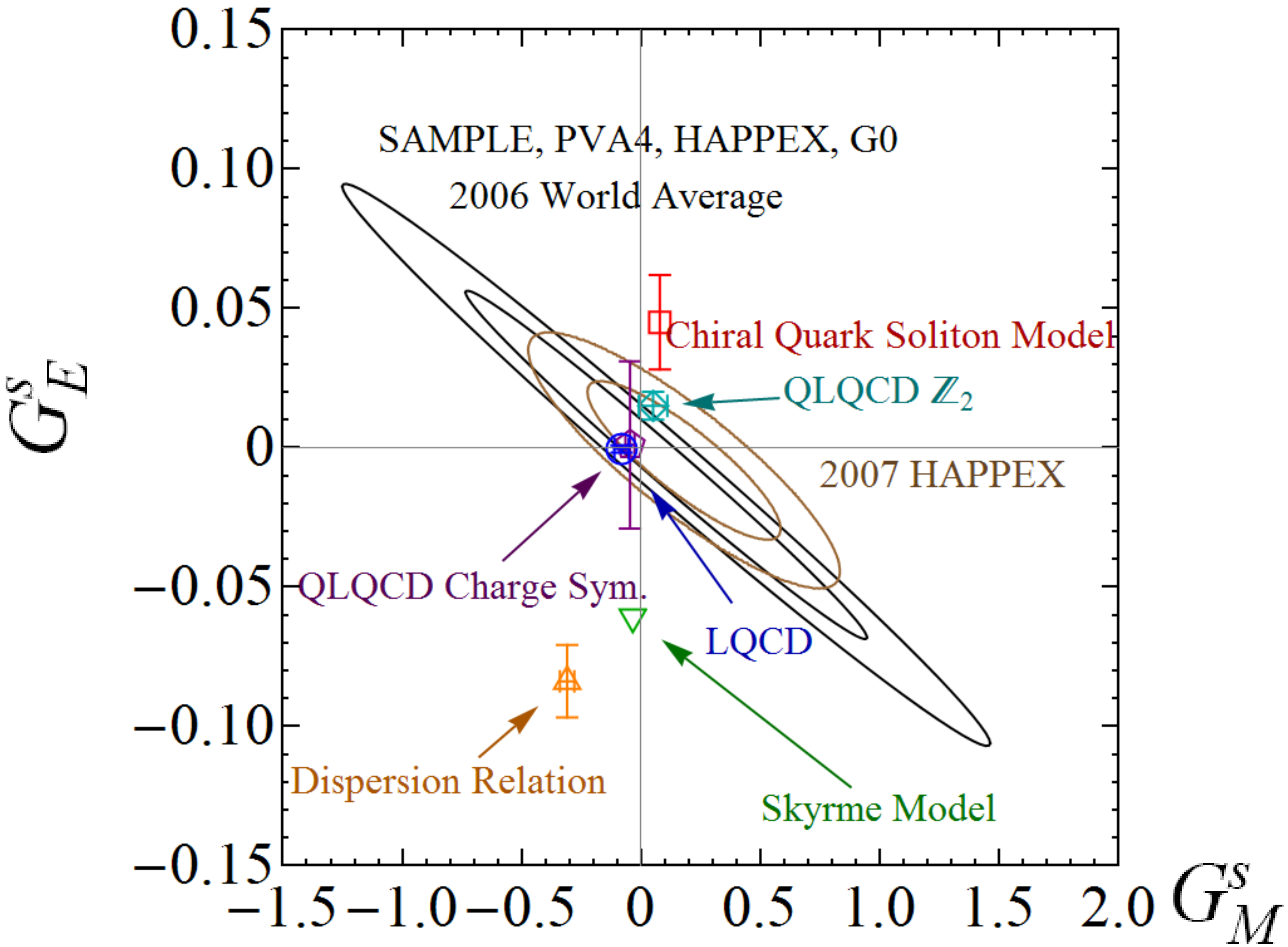
Preliminary

$$G_E^s(Q^2{=}0.1\text{ GeV}^2) = -0.00044(1)_{\text{stat}}(130)_{\text{rs}}$$

World Plot at $Q^2 \sim 0.1$ GeV^2

- G_E^s-G_M^s plots

HAPPEX: Phys.Rev.Lett.98:032301, 2007
SAMPLE, PVA4,HAPPEX,G0: Phys.Rev.Lett. 97 (2006) 102002



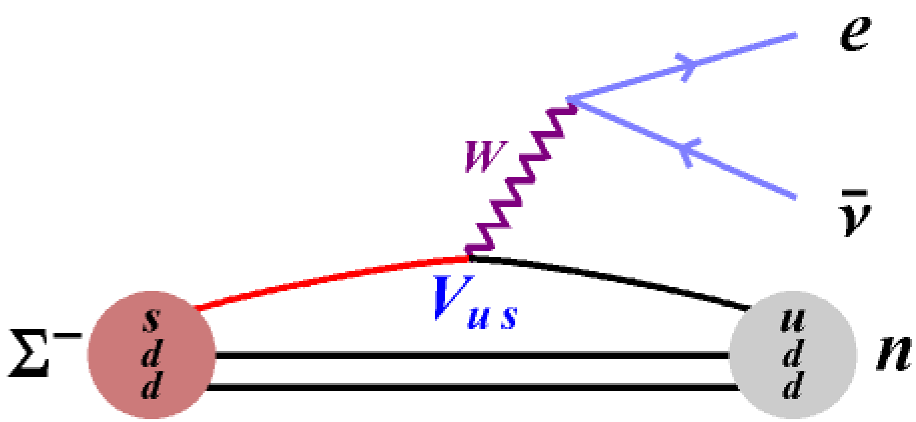
Hyperon Semileptonic Decays

in collaboration with

Kostas Orginos

Hyperon Decays

- Matrix element of the hyperon β-decay process $B_1 \to B_2 e^- \overline{\nu}$



$$\mathcal{M} = \frac{G_s}{\sqrt{2}} \overline{u}_{B_2} (O_\alpha^{\rm V} + O_\alpha^{\rm A}) u_{B_1} \overline{u}_e \gamma^\alpha (1+\gamma_5) v_\nu$$

with

$$O_\alpha^{\rm V} = f_1(q^2)\gamma^\alpha + \frac{f_2(q^2)}{M_{B_1}}\sigma_{\alpha\beta}q^\beta + \frac{f_3(q^2)}{M_{B_2}}q_\alpha$$

$$O_\alpha^{\rm A} = \left(g_1(q^2)\gamma^\alpha + \frac{g_2(q^2)}{M_{B_1}}\sigma_{\alpha\beta}q^\beta + \frac{g_3(q^2)}{M_{B_2}}q_\alpha\right)\gamma_5$$

SU(3) breaking

Hyperon Decay Experiments

- Experiments: CERN WA2, Fermilab E715, BNL AGS, Fermilab KTeV, CERN NA48

- Summary N. Cabibbo et al. 2003
  with f_2/f_1 and f_1 at the SU(3) limit

Decay	Rate (μs^{-1})	g_1/f_1	V_{us}
$\Lambda \to p e^- \bar{\nu}$	3.161(58)	0.718(15)	0.2224 ± 0.0034
$\Sigma^- \to n e^- \bar{\nu}$	6.88(24)	−0.340(17)	0.2282 ± 0.0049
$\Xi^- \to \Lambda e^- \bar{\nu}$	3.44(19)	0.25(5)	0.2367 ± 0.0099
$\Xi^0 \to \Sigma^+ e^- \bar{\nu}$	0.876(71)	1.32(+.22/−.18)	0.209 ± 0.027
Combined	—	—	0.2250 ± 0.0027

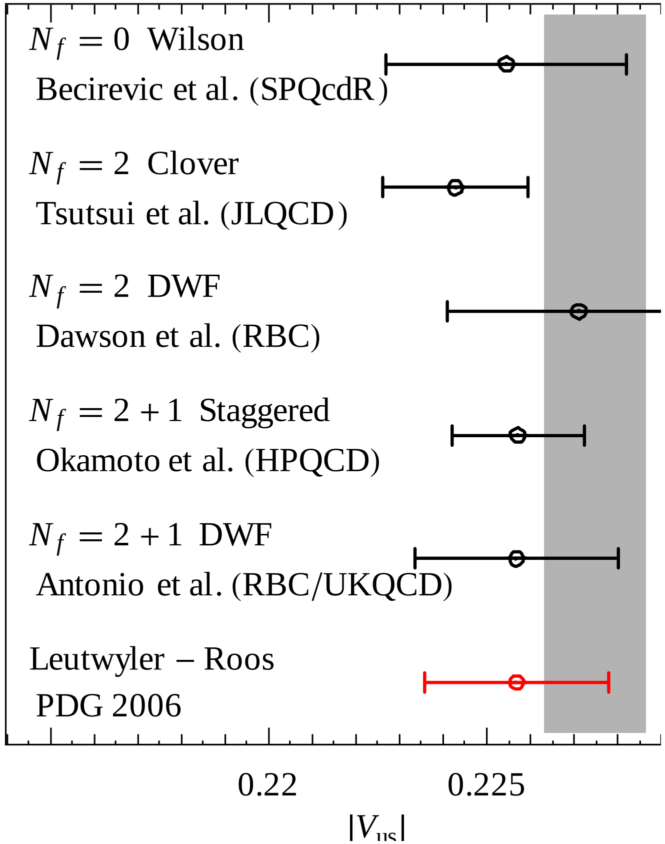
PDG 2006 number

- Better g_1/f_1 from lattice calculations?

$|V_{us}|$ from $K_{\ell 3}$ Decay

- Summary of $K_{\ell 3}$ calculations
  - Use common $|f_+ V_{us}| = 0.2169(9)$ from PDG 2006

Group	N_f	S_f	M_π (GeV)
SPQcdR	0	Wilson	0.500–1.000
JLQCD	2	Clover	0.440–0.960*
RBC	2	DWF	0.475–0.700
HPQCD	2+1	Staggered	0.500–0.700
RBC/UKQCD	2+1	DWF	0.390–0.700



$|V_{us}|$ from Leptonic Decays

- $K_{\mu 2}$ and $\pi_{\mu 2}$ decays

$$\left(\frac{|V_{us}|}{|V_{ud}|}\right)^2 = \left[\left(\frac{f_K}{f_\pi}\right)^2 \frac{M_K\left(1-m_\mu^2/M_K^2\right)^2}{M_\pi\left(1-m_\mu^2/M_\pi^2\right)^2}\left(1+\frac{\alpha}{\pi}\left(C_K-C_\pi\right)\right)\right]^{-1}\frac{\Gamma(K\to\mu\bar{\nu}_\mu)}{\Gamma(\pi\to\mu\bar{\nu}_\mu)}$$

- MILC collaboration (staggered, C. Aubin et al., 2004)

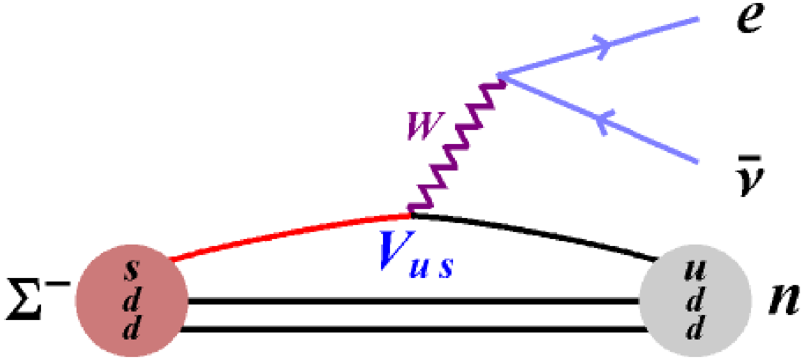
  f_K/f_π=1.210(4)(13)
- W. Marciano, 2004 $|V_{us}|$ = 0.2219(25)
- Other full-QCD f_K/f_π available since 2004

  RBC+UKQCD DWF: f_K/f_π = 1.24(2)

  MILC 2006: f_K/f_π=1.208(2)($^{+7}/_{-14}$)

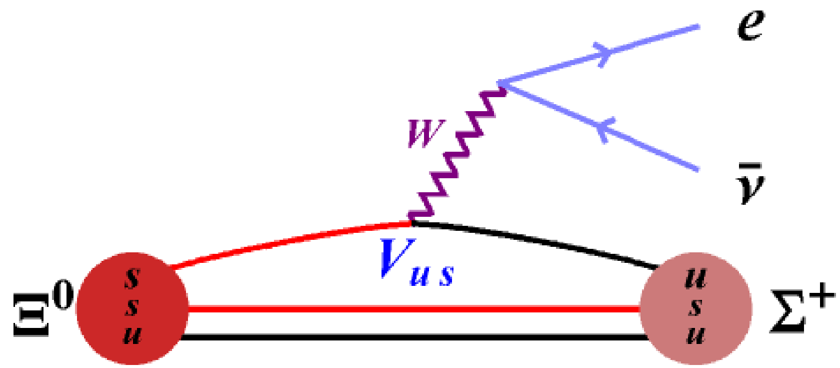
$|V_{us}|$ from Hyperons Decays

- Two quenched calculations, different channels



- Pion mass > 700 MeV
- $f_1(0) = -0.988(29)_{\text{stat.}}$
- $|V_{us}| = 0.230(5)_{\text{exp}}(7)_{\text{lat}}$

Guadagnoli et al.



- Pion mass ≈ 530–650 MeV
- $f_1(0) = 0.953(24)_{\text{stat}}$
- $|V_{us}| = 0.219(27)_{\text{exp}}(5)_{\text{lat}}$

Sasaki et al.

No systematic error estimate from **quenching** effects!

Ademollo-Gatto Theorem

- Chiral extrapolation:
  - SU(3) symmetry-breaking Hamiltonian

$$H' = \frac{1}{\sqrt{3}}\left(m_s - \frac{m_d + m_u}{2}\right)\bar{q}\lambda^8 q$$

  - There is no first-order correction $O(H')$ to $f_1(0)$; thus

$$f_1(0) = f_1^{SU(3)}(0) + O(H'^2)$$

  - Common choice of observable for H': $M_K{}^2 - M_\pi{}^2$
  - Step I: $R(M_K, M_\pi) = \dfrac{1 - |f'(0)|}{a^4(M_K^2 - M_\pi^2)^2}$
  - Step II: $R(M_K, M_\pi) = b_0 + b_1 a^2 (M_K^2 + M_\pi^2)$
- Obtain $|V_{us}|$ from

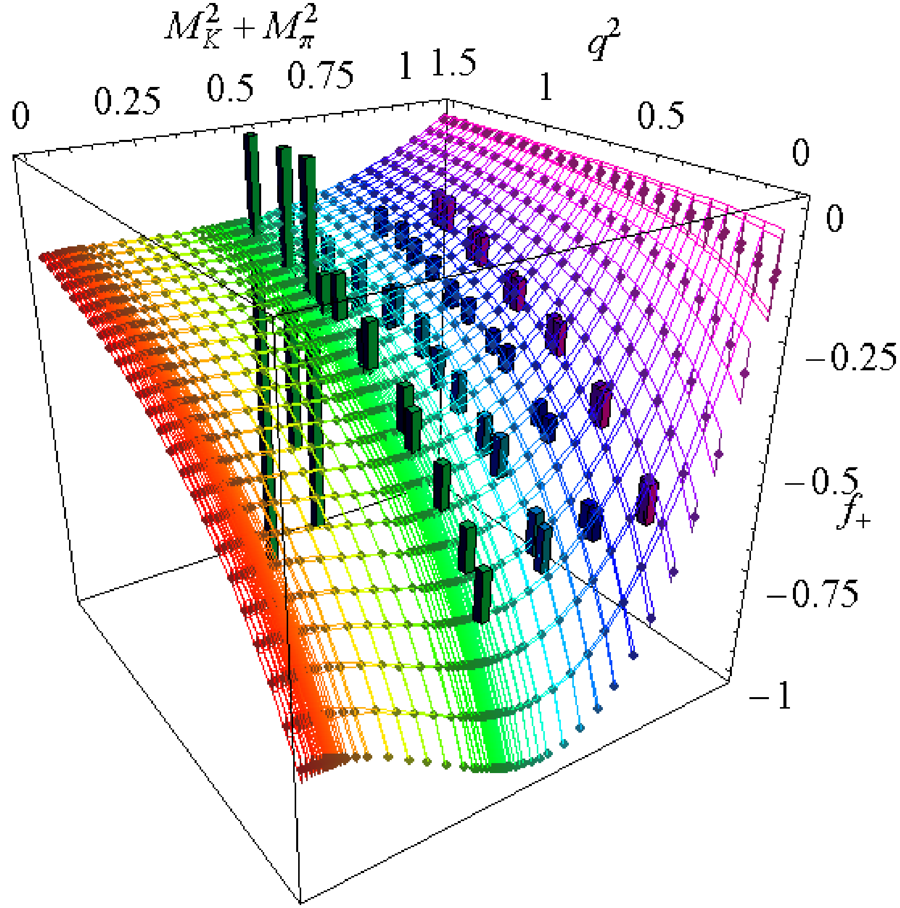
$$\Gamma = G_F^2 |V_{us}|^2 \frac{\Delta m^5}{60\pi^3}(1+\delta_{\rm rad}) \times \left[\left(1 - \frac{3}{2}\beta\right)\left(|f_1|^2 + |g_1|^2\right) + \frac{6}{7}\beta^2\left(|f_1|^2 + 2|g_1|^2 + \mathrm{Re}(f_1 f_2^*) + \frac{2}{3}|f_2^2|\right) + \delta_{q^2}\right]$$

with g_1/f_1 (exp) and f_2/f_1 (SU(3) value)

Simultaneous Fit

- $N_f = 2+1$ mixed action, $M_\pi \sim 350–600$ MeV
- Combined momentum and mass dependence

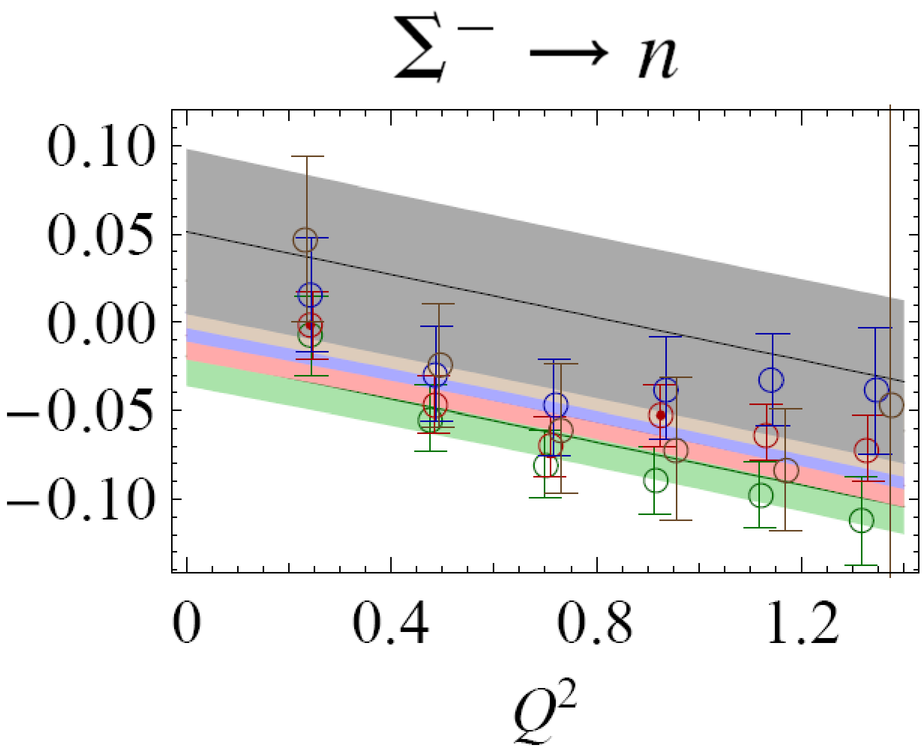
$$\Sigma^- \to n$$



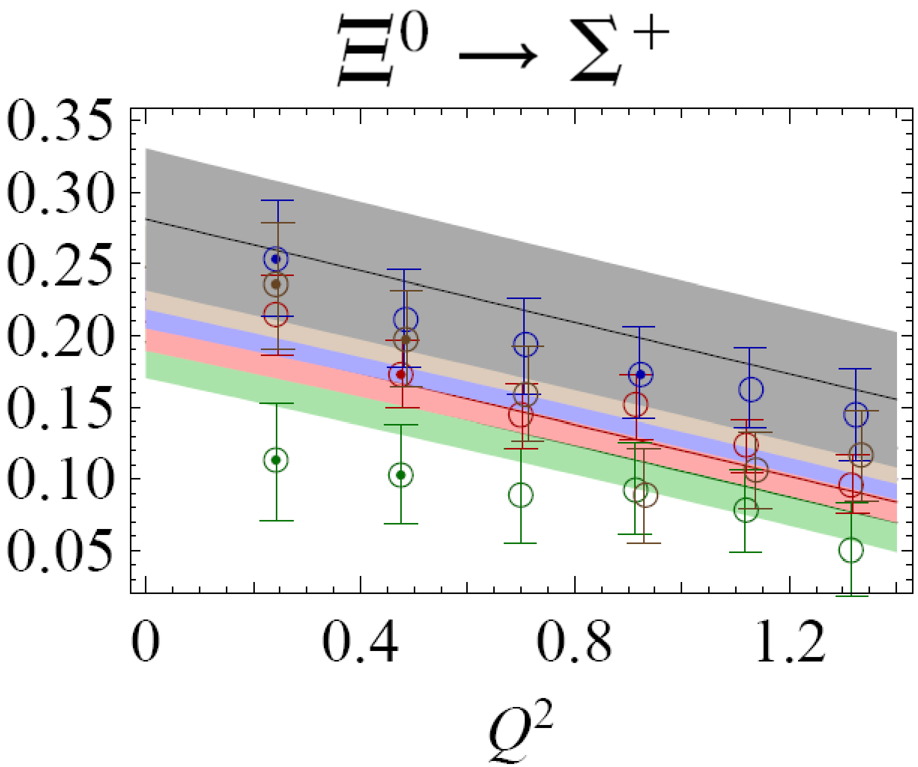
$f_1(0) = -0.95(3)$ (Preliminary)

Induced PS Form Factor f_3

- $N_f = 2+1$ mixed action, $M_\pi \sim 350$–600 MeV



$f_3(0) = 0.05(5)$

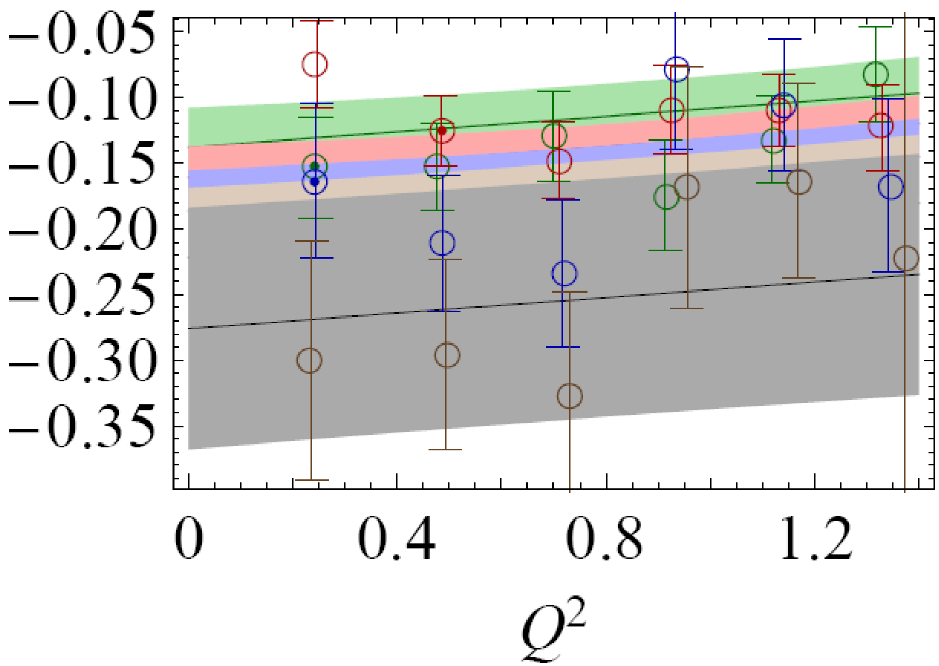


$f_3(0) = 0.28(5)$

Weak Electric Form Factor g_2

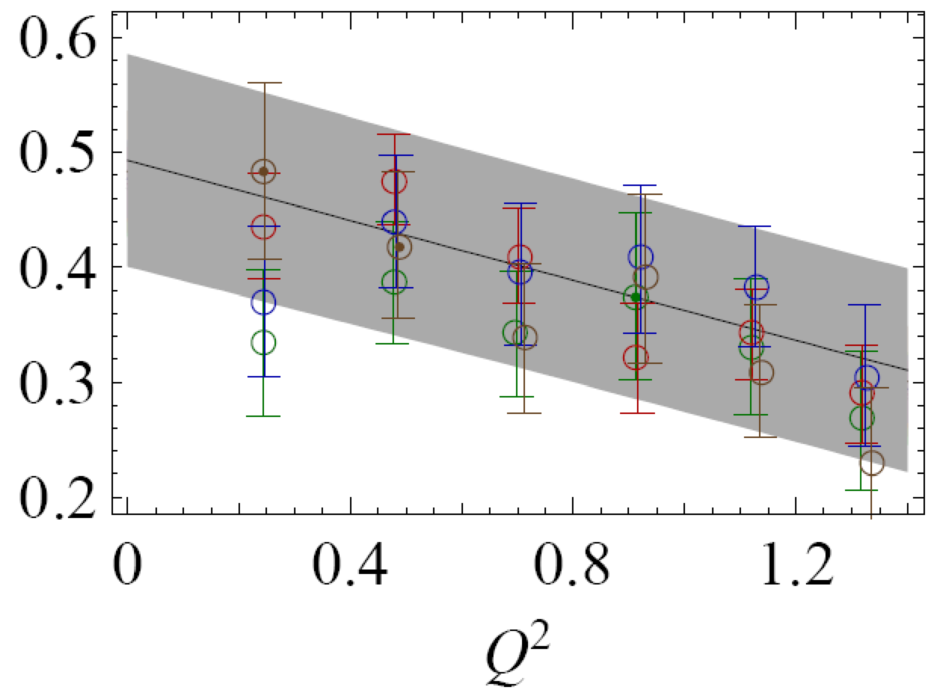
- $N_f = 2+1$ mixed action, $M_\pi \sim 350$–600 MeV

$\Sigma^- \to n$



$f_3(0) = -0.27(3)$

$\Xi^0 \to \Sigma^+$



$f_3(0) = 0.49(9)$

Summary and Outlook

- What we can provide at the moment ~ 300—350 MeV pion
  - Hyperon spectroscopy: spin predictions from mass ordering
  - Sigma and cascade axial coupling constants and form factors
  - Semileptonic decay form factors
  - Predictions for those quantities which cannot be measured
- In the near future…
  - Pion mass $\ll$ 300 MeV
  - Spectroscopy: orbital/radial excited states
  - Complete octet structure studies and decays, including Λ, Σ^0
  - Duplet-octet transition form factors
- Not enough time to mention…
  - Hadron interactions (π-π, π-K, N-N, N-Y scattering lengths)
  - Potentials and forces

Stay tuned on the latest lattice calculations