Measurements of $\pi\pi$ scattering lengths from $K^{\pm}$ decays Experimental studies of $K_{3\pi}$ and K_{e4} decays with the NA48/2 experiment at CERN

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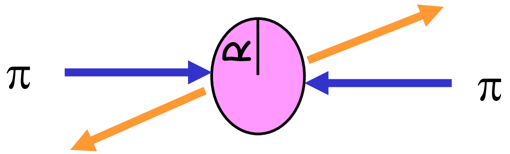
On behalf of the NA48/2 collaboration:

Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Wien

Outline

- what can we learn from $\pi\pi$ scattering lengths?
- the NA48/2 experimental setup
- measuring a_0, a_2 with:
  - cusp in $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$ decays
  - phase difference $\delta_{\pi\pi}$ in $K^{\pm} \rightarrow \pi^{\pm}\pi^{-}e^{\pm}\nu$ decays
- conclusion

$\pi\pi$ scattering lengths: why interesting?



angular wave number: $$k = \frac{\sqrt{2mE}}{\hbar}$$

- at low energy kR <<1 ⇨ S-wave dominates the total cross section
- Bose statistics ⇨ isospin I = 0, 2 allowed
- scattering matrix ⇨ $S|\pi\pi\rangle = \exp(2i\delta)|\pi\pi\rangle$

  i.e. 2 phases: $\delta_{0,2} = a_{0,2}\,k$ related to scattering lengths a_0, a_2

- NA48/2 uses charged kaons to measure scattering lengths a_0, a_2

  - cusp-effect in $K^{\pm} \rightarrow \pi^{\pm}\pi^0\pi^0$ decays
  - $\delta_{\pi\pi}$ and form factors in $K^{\pm} \rightarrow \pi^{\pm}\pi^{-}e^{+}\nu$ decays

  ⇨ study of $\pi\pi$ system near threshold

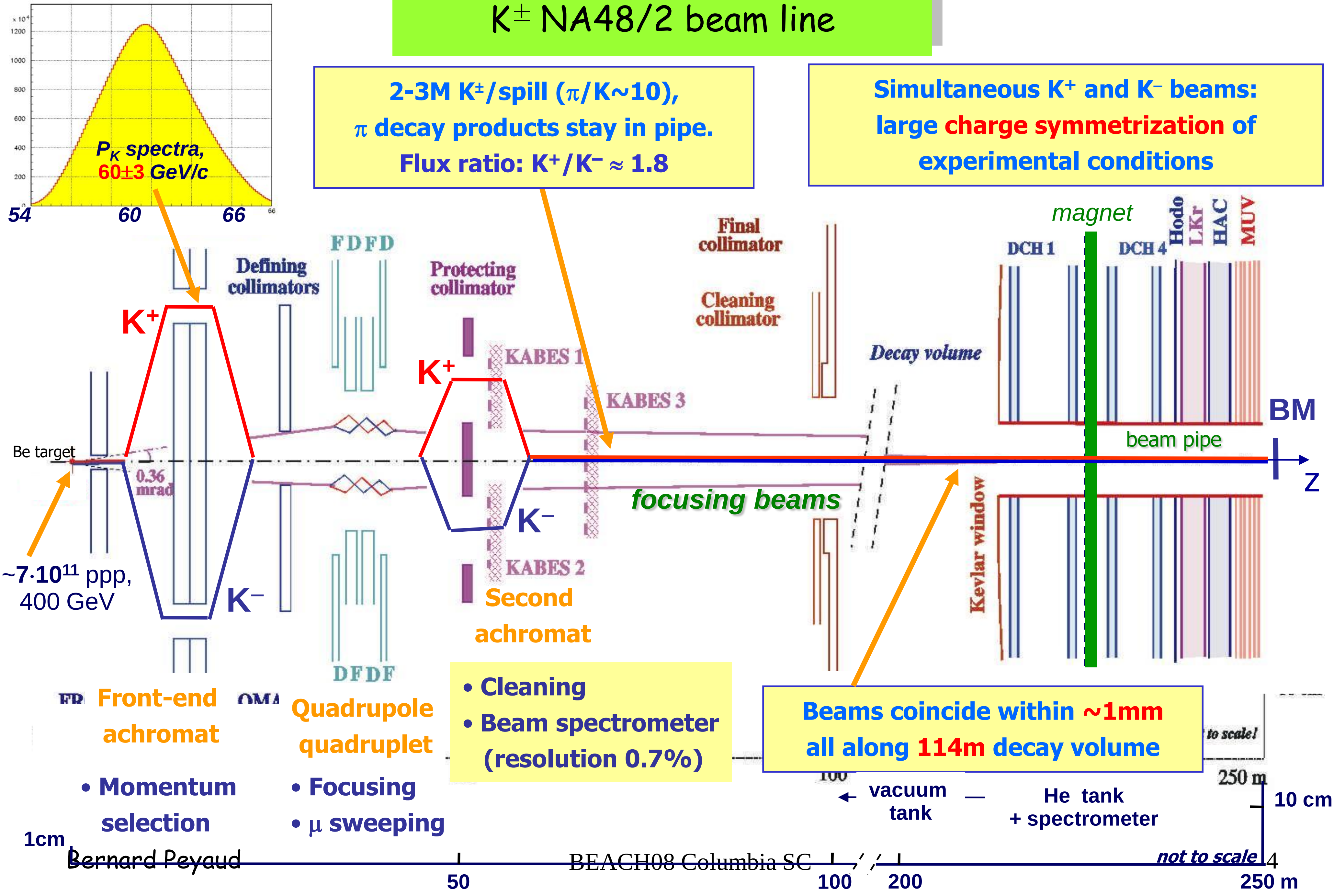
- *S*-wave scattering lengths a_0, a_2 are essential parameters of χPT

  spontaneous symmetry breaking from $\langle q\bar{q}\rangle$ condensate ?

$K^{\pm}$ NA48/2 beam line

2-3M $K^{\pm}$/spill ($\pi/K \sim 10$),
π decay products stay in pipe.
Flux ratio: $K^{+}/K^{-} \approx 1.8$

Simultaneous K^{+} and K^{-} beams:
large charge symmetrization of experimental conditions



~$7 \cdot 10^{11}$ ppp,
400 GeV

Second achromat

Front-end achromat
- **Momentum selection**

Quadrupole quadruplet
- **Focusing**
- **μ sweeping**

- **Cleaning**
- **Beam spectrometer (resolution 0.7%)**

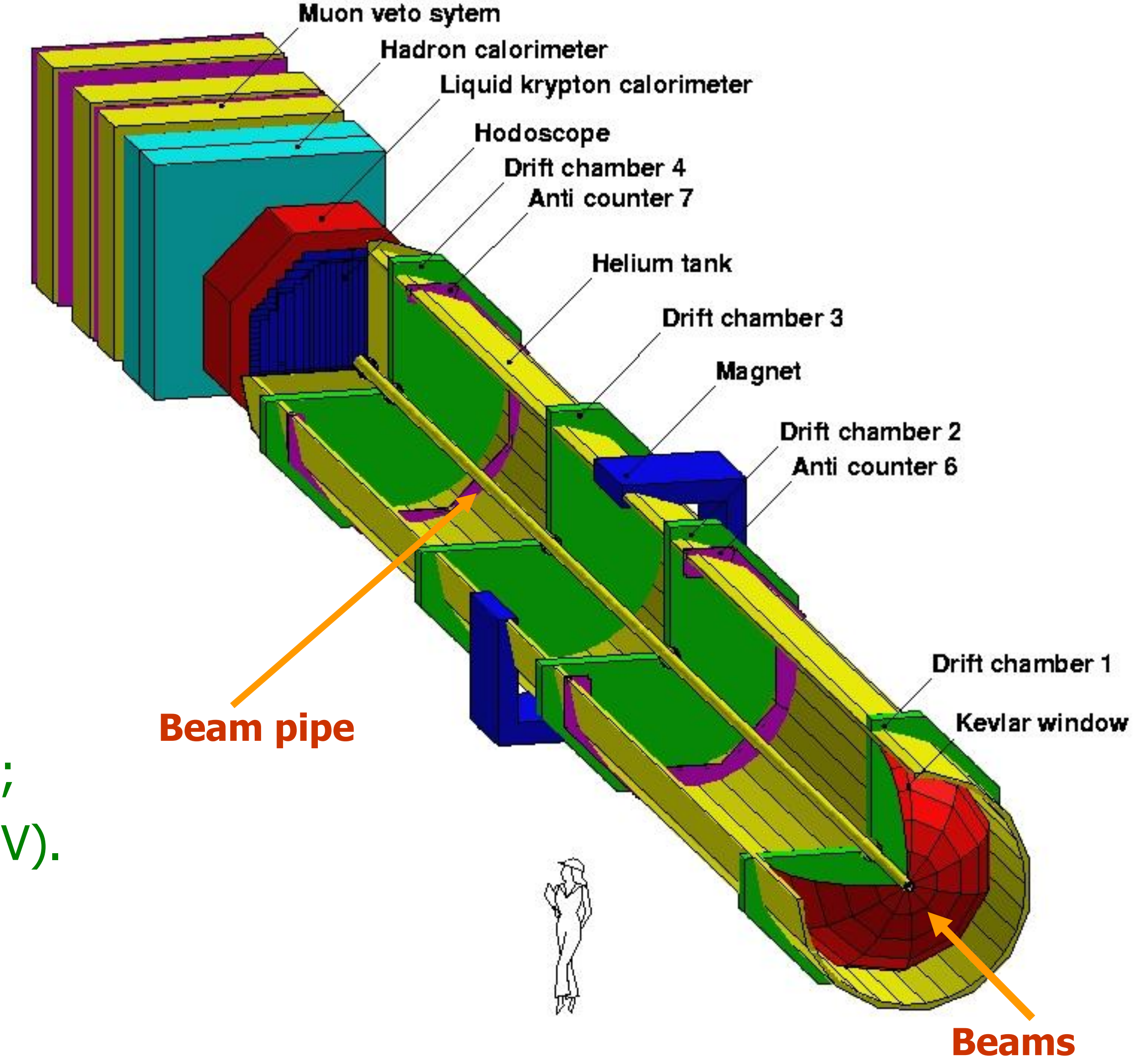
Beams coincide within ~1mm all along 114m decay volume

vacuum tank
He tank + spectrometer

The NA48 detector

Main detector components:

- Magnetic spectrometer (4 DCHs):
  4 views/DCH: redundancy ⇒efficiency;
  used in trigger logic;
  $\Delta p/p = 1.0\% + 0.044\%{*}p$ [GeV/c].
- Hodoscope
  fast trigger;
  precise time measurement (150ps).
- Liquid Krypton EM calorimeter (LKr)
  High granularity, quasi-homogenious;
  $\sigma_E/E = 3.2\%/E^{1/2} + 9\%/E + 0.42\%$ [GeV];
  $\sigma_x=\sigma_y=0.42/E^{1/2} + 0.6$mm (1.5mm@10GeV).
- Hadron calorimeter, muon veto counters,
  photon vetoes.



NA48 data

Prime goal of NA48/2:

measurement of CP-violating charge asymmetry in $K^\pm \to 3\pi$ decays

both modes with large BR's of (2-5) 10^{-2}


View of the NA48/2 beam line

2003 run: ~ 50 days

2004 run: ~ 60 days

Total statistics in 2 years:

$K^\pm \to \pi^\pm\pi^-\pi^+$: ~$4\cdot10^9$ evts

$K^\pm \to \pi^\pm\pi^0\pi^0$: ~$1\cdot10^8$ evts

Rare $K^\pm$ decays with BR's down to 10^{-9} also measured and compared to χPT predictions

⇨ cusp in $K^\pm \to \pi^\pm\pi^0\pi^0$ decays

⇨ $\delta_{\pi\pi}$ in K_{e4} decays

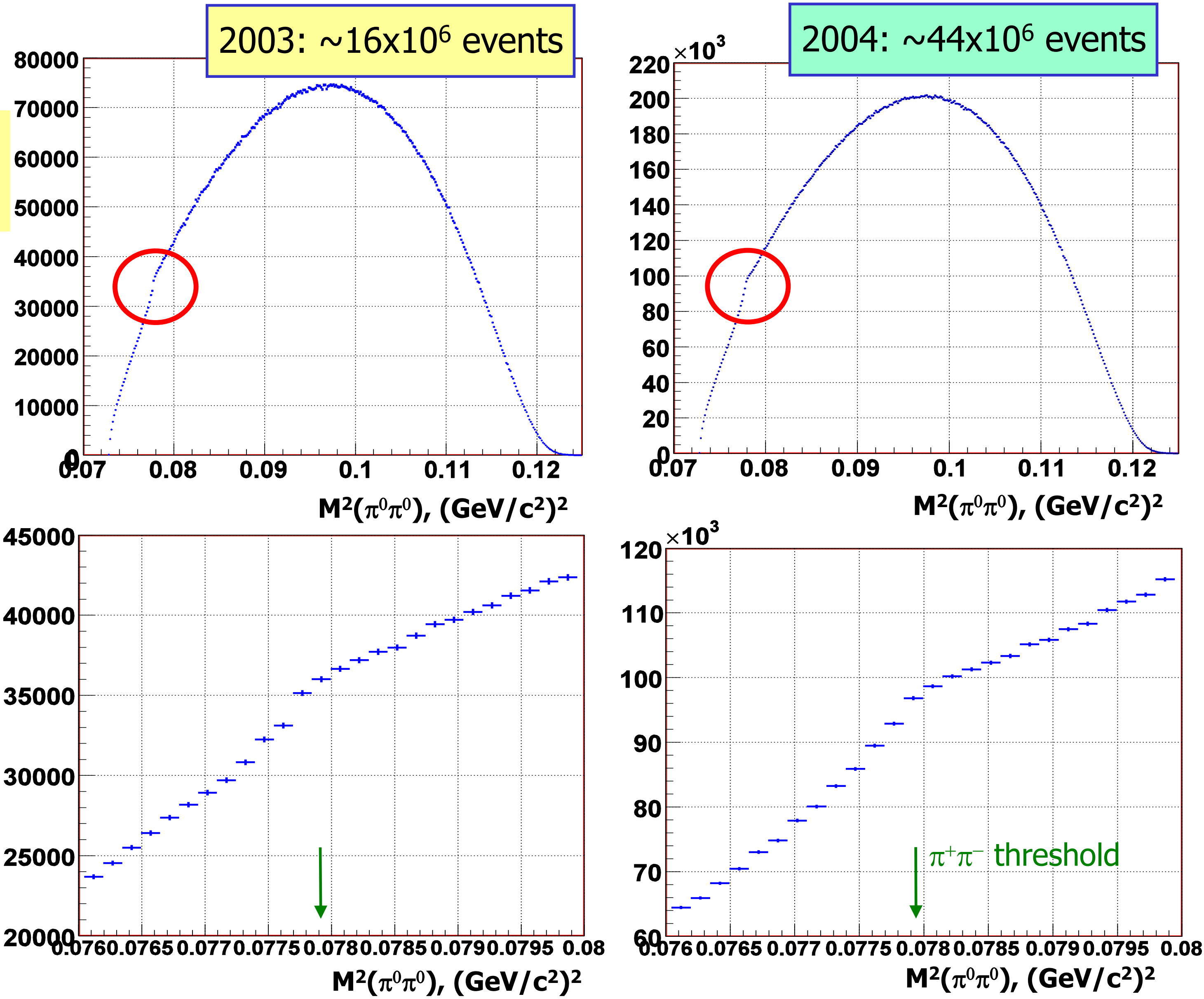
>200 TB of data recorded

Evidence of cusp structure in $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ decays

First observation of the cusp made with 2003 data

Addition of 2004 data: statistics x 3.7

Homogeneous selection conditions + MC/Data statistics allows global analysis of all data

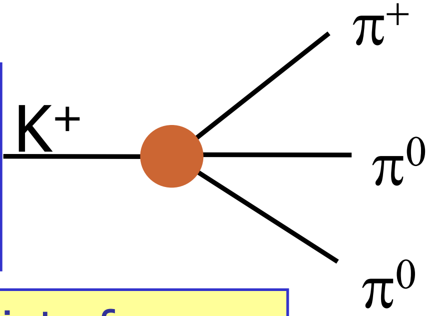

2003: ~16x10^6 events

2004: ~44x10^6 events

$\pi^+\pi^-$ threshold

Theory: final state rescattering. N. Cabibbo, PRL 93 (2004) 121801

$$\mathcal{M}(K^{\pm}\rightarrow\pi^{\pm}\pi^{0}\pi^{0}) = \mathcal{M}_0 + \mathcal{M}_1$$



Kaon rest frame:

$u = 2m_K(m_K/3 - E_{odd})/m_\pi^2$

$v = 2m_K(E_1 - E_2)/m_\pi^2$

Direct emission (k′,h′ << g):

$$\mathcal{M}_0 = A_0(1+g_0u/2+h'_0u^2/2+k'_0v^2/2)$$

$$\mathcal{M}_+ = A_+(1+g_+u/2+h'_+u^2/2+k'_+v^2/2)$$

Rescattering amplitude:

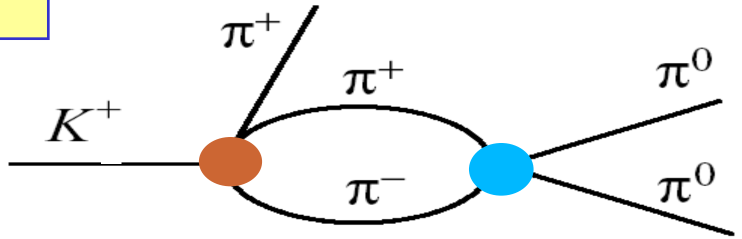
$$\mathcal{M}_1 = -2/3(a_0-a_2)m_+\mathcal{M}_+ \sqrt{1-\left(\frac{M_{00}}{2m_+}\right)^2}$$

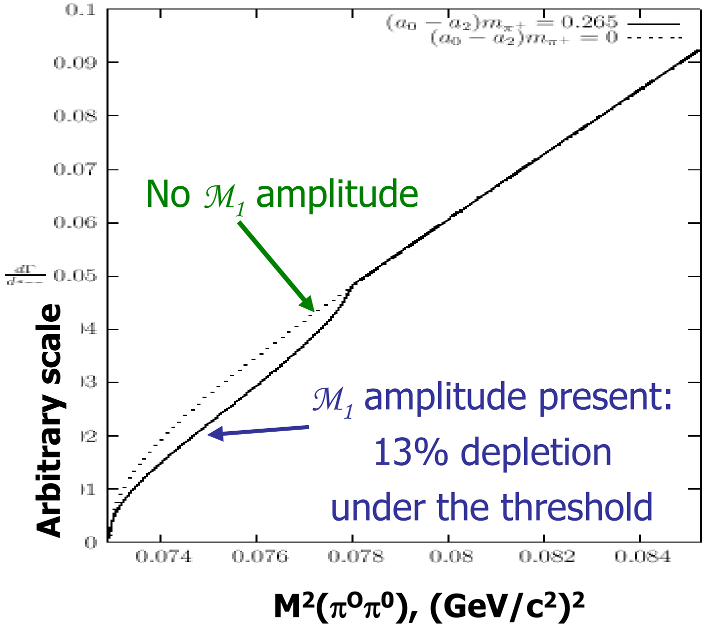
Negative interference under threshold

Combination of S-wave ππ scattering lenghts

K± → 3π± amplitude at threshold

(isospin symmetry assumed here)





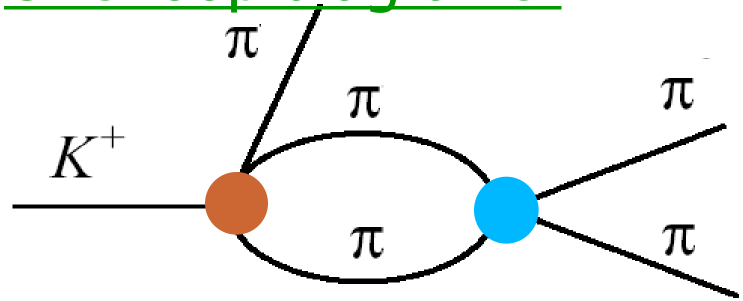
No $\mathcal{M}_1$ amplitude

$\mathcal{M}_1$ amplitude present: 13% depletion under the threshold

Theory: 2 loops diagrams: N. Cabibbo and G. Isidori (CI), JHEP 503 (2005) 21

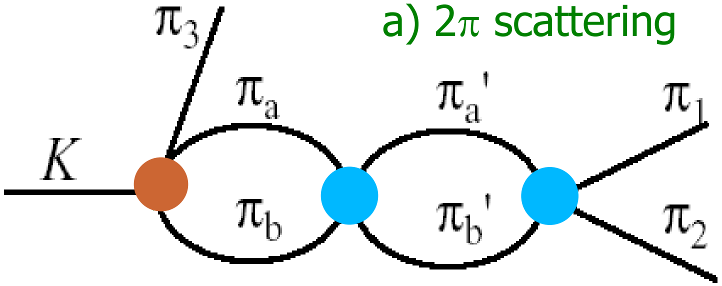
- S-wave scattering lengths (a_x, a_{++}, a_{+-}, a_{+0}, a_{00}) are linear combinations of a_0,a_2
- Isospin symmetry breaking following J. Gasser.
- Radiative corrections missing; (a_0–a_2) precision ~5%
- *V-dependent terms ~ (k'/2)V^2 introduced recently in "unperturbed" $K^{\pm}\rightarrow\pi^{\pm}\pi^{+}\pi^{-}$ and $K^{\pm}\rightarrow\pi^{\pm}\pi^{0}\pi^{0}$ amplitudes.*

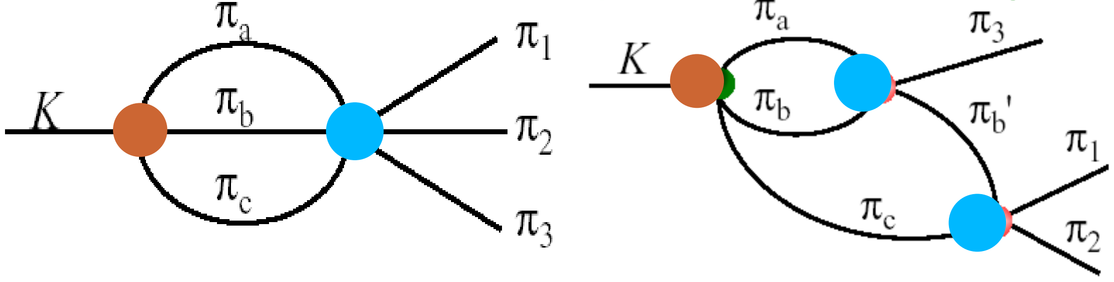
One-loop diagrams:



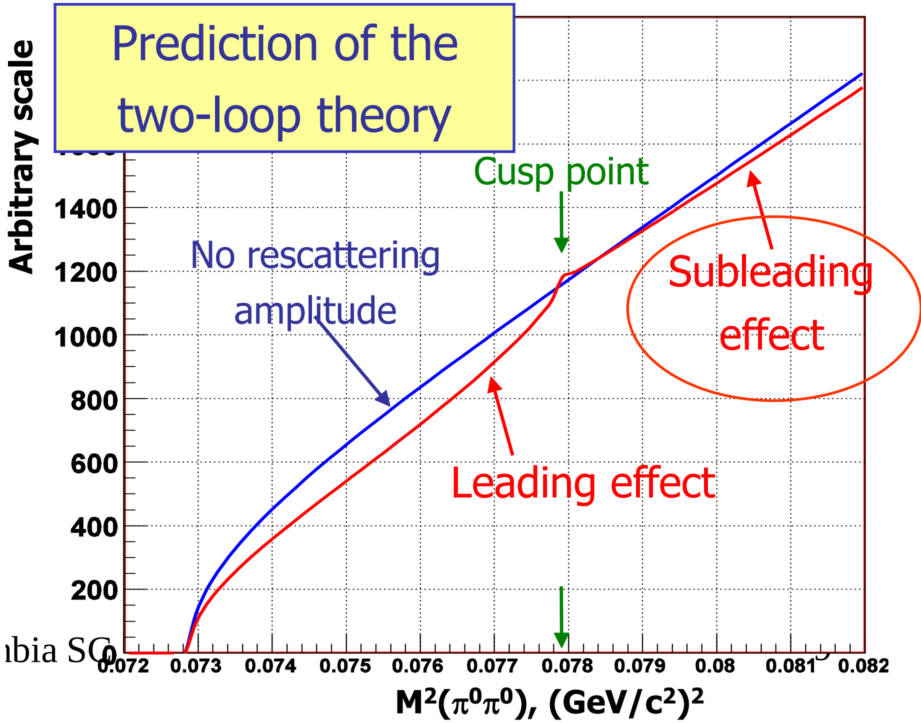
Two-loop diagrams:

a) 2π scattering




b) irreducible 3π scattering

c) reducible 3π scattering


Prediction of the two-loop theory

Theory: effective field

G. Colangelo, J. Gasser, B. Kubis, A. Rusetsky (CGKR) Phys.Lett. B638 (2006) 187-194

•polynomial parts of amplitudes expressed in terms of (U,V)-slopes

•g,h,k numerically different from CI ones, and faster integration of the amplitude over V.

- Non-relativistic Lagrangian for effective fields
- Validity in the whole decay region.
- different part of amplitude (wrt CI) absorbed in the polynomial terms (⇨ different correlations).
- At two loop level different formulae for amplitude
- FORTRAN code written by authors

- CI fit uses our recently measured parameters of the **$K^\pm \to 3\pi^\pm$** amplitude

$$\mathcal{M}_+ = A_+(1+g_+u/2+h'_+u^2/2+k'_+v^2/2):$$

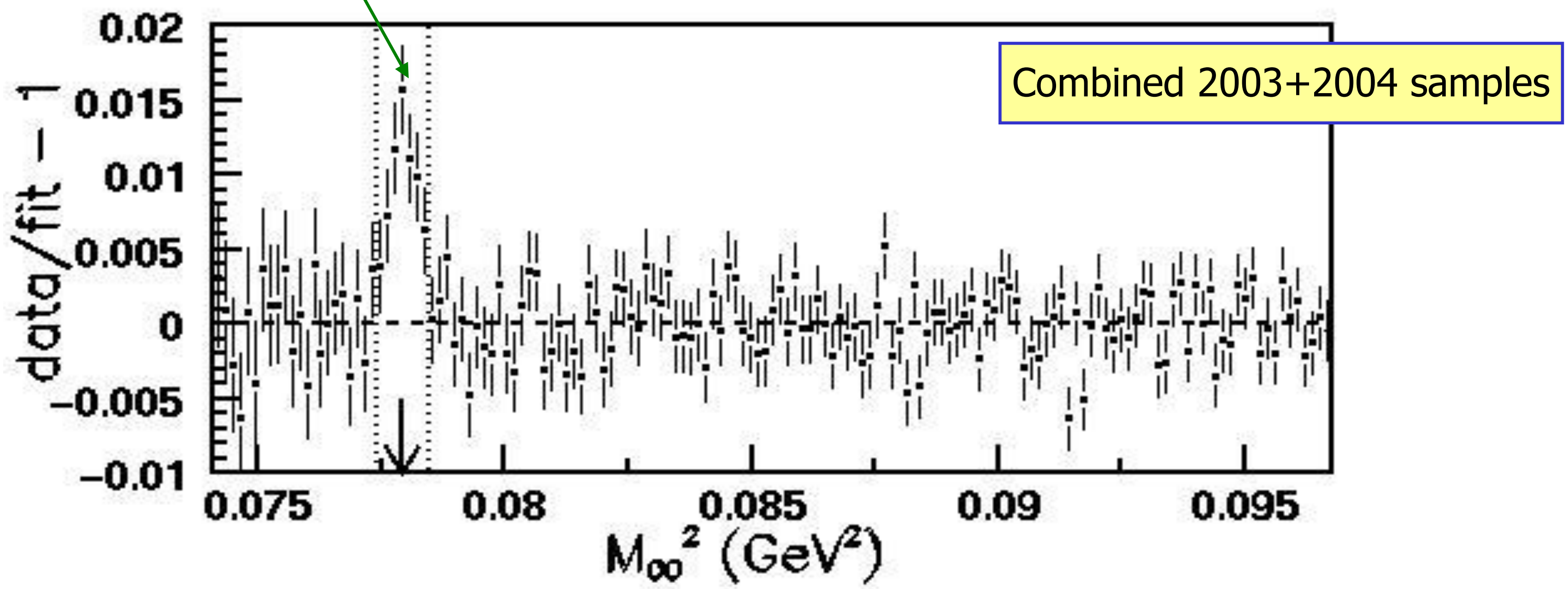
$$g_+ = -\,0.21117(15); \quad h'_+ = 0.00671(26); \quad k'_+ = -\,0.00477(8)$$

- CGKR has similar parameters but numerically different due to additional rescattering terms in the in M_+ amplitude.

⇨ simultaneous fit of the NA48/2 **$K^\pm \to 3\pi^\pm$** and **$K^\pm \to \pi^\pm\pi^0\pi^0$** Dalitz plots for CGKR fit:

$$g_+ = -\,0.1837; \quad h'_+ = 0.00043; \quad k'_+ = -\,0.0059$$

Pionium signature


7 Points around $\pi^+\pi^-$ threshold are excluded from the fit due to absence of EM corrections in the model

Combined 2003+2004 samples

Excess of events in the excluded interval (CI fit),

if interpreted as due to pionium decaying as $A_{2\pi}\rightarrow\pi^0\pi^0$,

gives $R=\Gamma(\mathbf{K}^\pm\rightarrow\pi^\pm A_{2\pi})/ R=\Gamma(\mathbf{K}^\pm\rightarrow\pi^\pm \pi^+\pi^-) = (1.8\pm0.3)\times10^{-5}$.

⇒ Prediction [Z.K. Silagadze, JETP Lett. 60 (1994) 689]: $R=0.8\times10^{-5}$.

Uncertainties & results (1)

Experimental constants are needed in the amplitude calculation

“external” uncertainty dominated by $R = (A_{++-}/A_{+00})|_{\text{threshold}} = 1.975 \pm 0.015$

Using a chiral symmetry constraint [Colangelo et al., PRL 86 (2001) 5008]:

$$a_2 = (-0.0444 +/- 0.0008) + 0.236(a_0-0.22) - 0.61(a_0-0.22)^2 - 9.9(a_0-0.22)^3$$

$$\text{CI}: (a_0-a_2)m_+ = 0.268 \pm 0.003_{\text{stat.}} \pm 0.002_{\text{syst.}} \pm 0.001_{\text{ext.}}$$

$$\text{CGKR}: (a_0-a_2)m_+ = 0.266 \pm 0.003_{\text{stat.}} \pm 0.002_{\text{syst.}} \pm 0.001_{\text{ext.}}$$

Theory precision uncertainty for CI case: $\delta(a_0-a_2)m_+ = 0.013$.

(preliminary)

Uncertainties and results (2)

With both a_0 and a_2 as free parameters in the fit

(preliminary)

CI case:

$$(a_0-a_2)m_+ = 0.266 \pm 0.005_{\mathrm{stat.}} \pm 0.002_{\mathrm{syst.}} \pm 0.001_{\mathrm{ext.}}$$
$$a_2 m_+ = -0.039 \pm 0.009_{\mathrm{stat.}} \pm 0.006_{\mathrm{syst.}} \pm 0.002_{\mathrm{ext.}}$$

Theoretical uncertainty: 5%.

CGKR case:

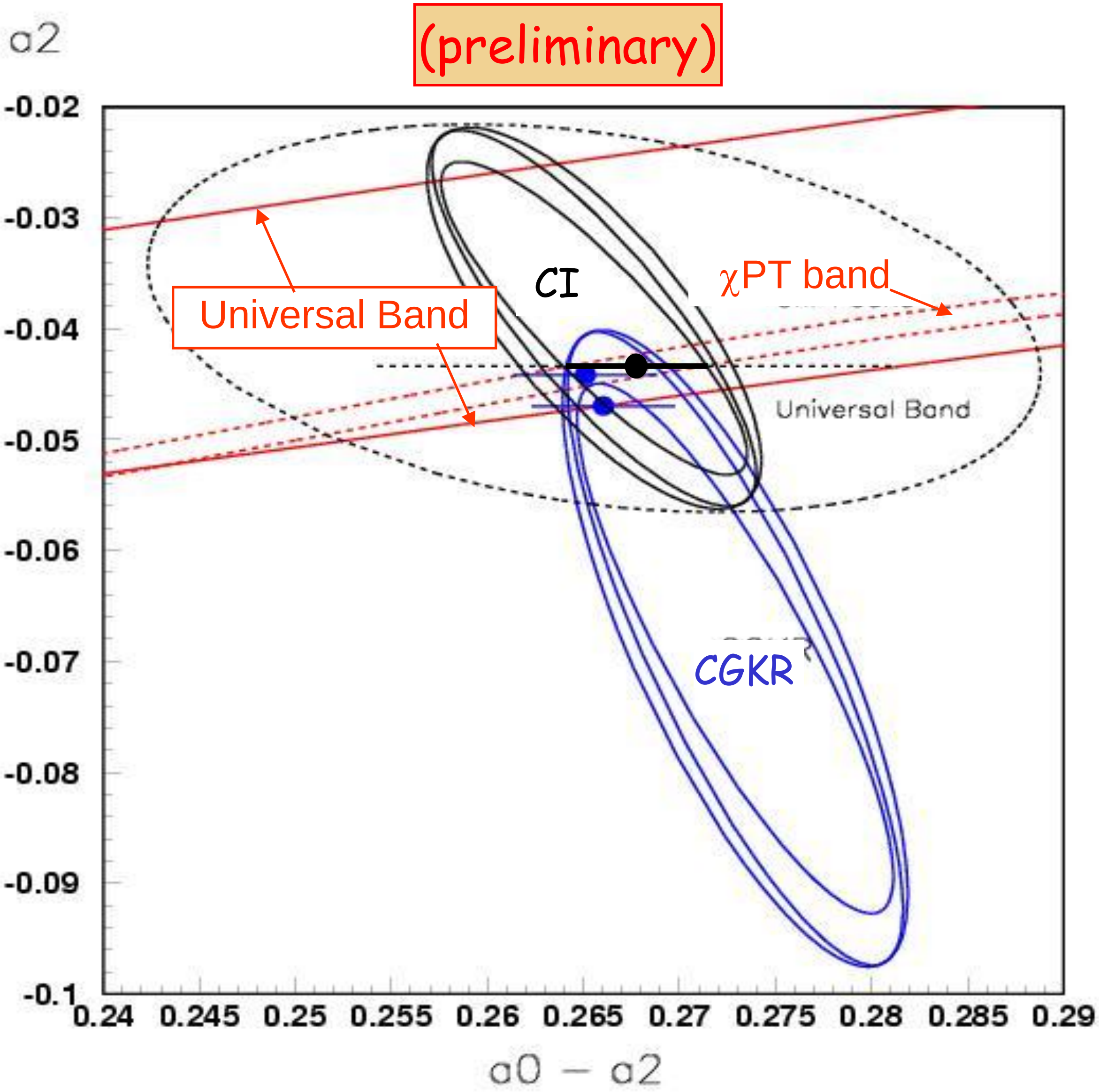
$$(a_0-a_2)m_+ = 0.273 \pm 0.005_{\mathrm{stat.}} \pm 0.002_{\mathrm{syst.}} \pm 0.001_{\mathrm{ext.}}$$
$$a_2 m_+ = -0.065 \pm 0.015_{\mathrm{stat.}} \pm 0.010_{\mathrm{syst.}} \pm 0.002_{\mathrm{ext.}}$$

Theoretical uncertainty not given.

CGKR case has large correlations between a_2 and polynomial terms of amplitude.

CI parameters very close to PDG’s, a_0,a_2 better disentangled from g and h slopes.

NA48/2: cusp fit results



— theoretical limits (Universal Band)

— 68.3% prob. CI fit results:
·stat. error only
·stat. + syst. error
·stat. + syst. + external
·all including theoretical error

—●— CI fit with χPT constraint

— 68.3% prob. CGKR fit results:
-statistical only error;
-statistical and systematical;
-stat., syst. and external;

—●— CGKR fits with χPT constraint and lower limit of Universal Band constraint

K_{e4} ($K^{\pm} \rightarrow e^{\pm} \nu \pi^{+}\pi^{-}$)

- Results from 2003 Data analysis published (EPJC 54 3 (2008) 411)
- Event selection, reconstruction and FF extraction
- K_{e4} decays : kinematic variables and Form Factors
- Understanding $\delta_{\pi\pi}$ in terms of $\pi\pi$ scattering lengths

- the $\pi\pi$ scattering process is described theoretically by dispersion relations (Roy equations) that relate amplitudes with different isospin. As a result $\delta_{\pi\pi}$ depends essentially on two parameters, the scattering lengths a_0^0 and a_0^2 .

- at low energy χPT constrains a_0^0 and a_0^2 through the size of the quark condensate

Ke4 decays: event selection and background rejection

Topology of $\pi^+\pi^-e^{\pm}\nu$ signal : 3 charged tracks , 2 opposite sign pions, 1 electron (LKr info E/p ~ 1), some missing energy and p_T (neutrino)

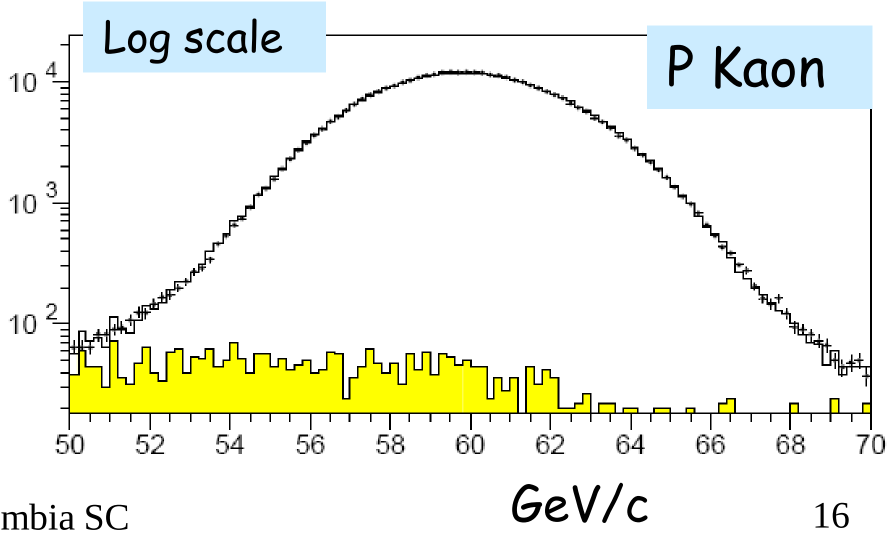
Background : main sources
$\pi^{\pm}\pi^+\pi^-$ decay followed by $\pi \rightarrow e\nu$ (dominant) or π misidentified as e
$\pi^{\pm}\pi^0(\pi^0)$ decay + π^0 Dalitz decay ($e^+e^-\gamma$) + e misidentified as π and γ (s) undetected

Total background level is kept at ~ 0.5 % relative level

Control from data sample : Wrong Sign

-total charge ($\pm$1) as Right Sign events
-<u>electron charge opposite to total charge</u>

In RS events:
twice the rate if coming from $K3\pi$
same rate if coming from $K2\pi(\pi0)$



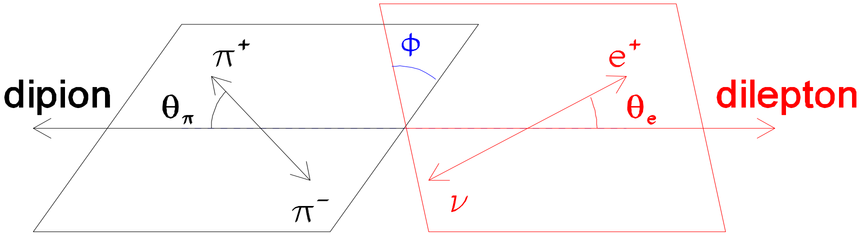
Ke4 charged decays : 4-body decay formalism

Five kinematic variables

(Cabibbo-Maksymowicz):

2 invariant masses: $S_\pi = M^2_{\pi\pi}$, $S_e = M^2_{e\nu}$

3 angles : $\cos\theta_\pi$, $\cos\theta_e$ and ϕ.



partial wave expansion of the amplitude:

F, G = Axial Form Factors

$F = F_s e^{i\delta s} + F_p e^{i\delta p} \cos\theta_\pi +$ d-wave term…

$G = G_p e^{i\delta g} +$ d-wave term…

H = Vector Form Factor

$H = H_p e^{i\delta h} +$ d-wave term…

expansion in powers of q^2 , $Se/4m\pi^2$
$(q^2 = (S_\pi / 4m_\pi^2 - 1))$

$$F_s = f_s + f_s' q^2 + f_s'' q^4 + f_e \left(S_e / 4m_\pi^2 \right) + ..$$

$$F_p = f_p + f_p' q^2 + ..$$

$$G_p = g_p + g_p' q^2 + ..$$

$$H_p = h_p + h_p' q^2 + ..$$

In each $m_{\pi\pi}$ bin the fit parameters are F_s , F_p , G_p , H_p and $\delta = \delta_s - \delta_p$

Ke4 charged decays : Total Data sample (2003 + 2004) = 1.15x10^6 events !

The 5 dimensional space spanned by C.M. variables, ($M_{\pi\pi}$, $M_{e\nu}$, cosθ_π, cosθ_e and ϕ) is binned with 10x5x5x5x12=15000 boxes with equal population

The set of Form Factor values is used to minimize a log-likelihood estimator taking into account the statistics of the data and MC in each box.

K^+ sample (739 500 events)	49 events/box
K^- sample (411 600 events)	27 events/box
MC K+ sample (17.7 Millions events)	~1180 events/box
MC K- sample (9.8 Millions events)	~653 events/box

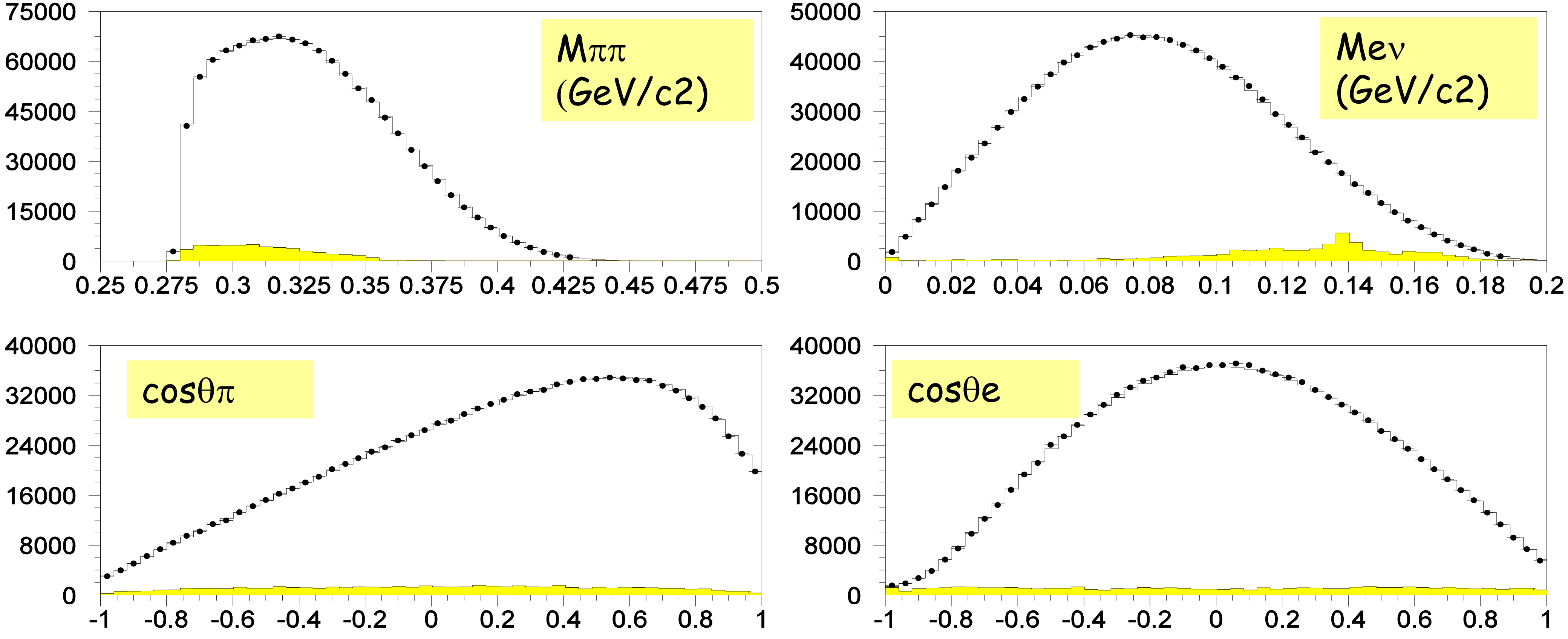
Ratio K+/K- ~ 1.8 both in Data and MC (run by run basis)
Ratio MC/Data ~ 24. both for K+ and K- (run by run basis)

Ke4 charged decays : the mass and cosθ distributions

K^+ and K^- samples fitted separately, results combined

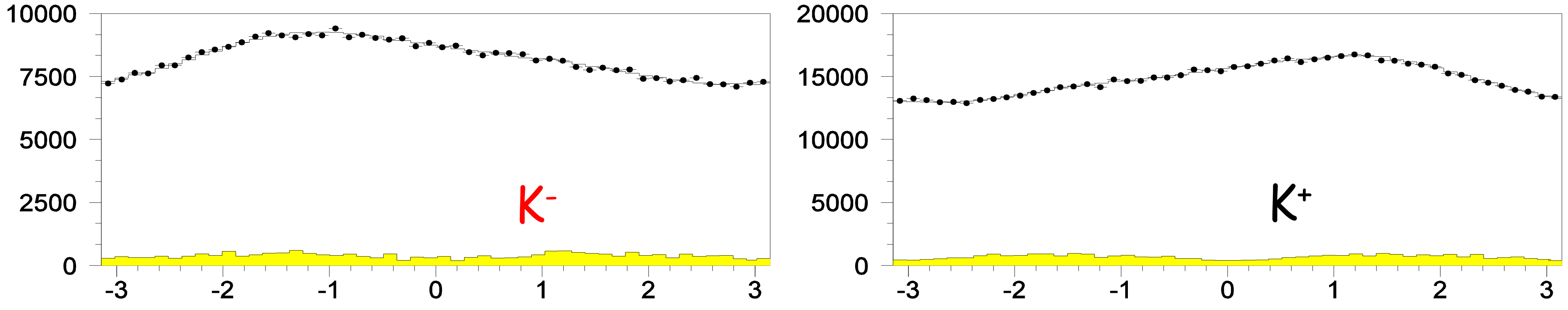
data (symbols), simulation after fit (hist) and background (× 10 to be visible)

Background flat in $\cos\theta_\pi$, $\cos\theta_e$, ϕ, concentrated at low $M_{\pi\pi}$ and $M_{e\nu} = m_\pi$



Ke4 charged decays : the ϕ distributions

CP symmetry : $[d^5N(K^+)]$ at θ_π,θ_e,ϕ = $[d^5N(K^-)]$ at $\theta_\pi,\theta_e,-\phi$



ϕ (rad)

Ratio K+/K- = 1.8 as expected from beam composition

Ke4 joint distribution ($\cos\theta_e,\phi$) à la Pais-Treiman

Distribution has 9 intensity functions: $I_i = I_i(m_{\pi\pi}, m_{e\nu})$

$$\mathbf{N}(\cos\theta_e,\phi) = I_1+I_2\cos 2\theta_e + I_3\sin^2\theta_e\cos 2\phi$$
$$+I_4\sin 2\theta_e\cos\phi + I_5\sin\theta_e\cos\phi + I_6\cos\theta_e$$
$$+I_7\sin\theta_e\sin\phi + I_8\sin 2\theta_e\sin\phi + I_9\sin^2\theta_e\sin 2\phi$$

With

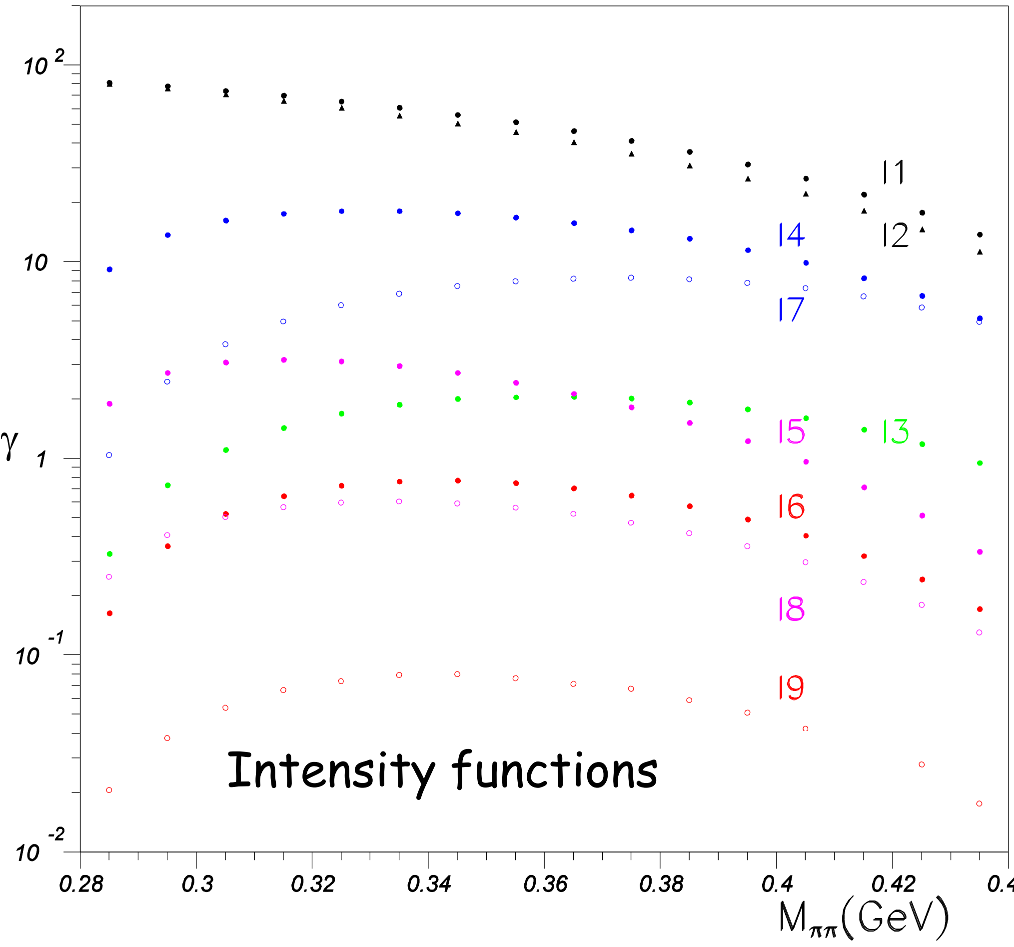
$$I_1 = \mathbf{f_s}^2\gamma^2 + f_p^{\,2}\gamma^2/3 + \mathbf{g}^2(\beta^2+\alpha^2/3) + \mathbf{h}^2\beta^2\gamma^2 + 2/3\,\mathbf{f_p}\,\mathbf{g}\cos\varepsilon_f\,\alpha\gamma$$
$$I_2 = -\mathbf{f_s}^2\gamma^2 + \mathbf{g}^2/3\,(\beta^2-\alpha^2) - f_p^{\,2}/3\,\gamma^2 + \mathbf{h}^2/3\,\beta^2\gamma^2 - 2/3\,\mathbf{f_p}\,\mathbf{g}\cos\varepsilon_f\,\alpha\gamma$$
$$I_3 = -2/3\,\mathbf{g}^2\beta^2 + 2/3\,\mathbf{h}^2\beta^2\gamma^2$$

$I_4 = \pi/2\,\mathbf{f_s}\,\mathbf{g}\cos(\delta)\,\beta\gamma$ $\quad$ $\boxed{I_7 = \pi\,\mathbf{f_s}\,\mathbf{g}\sin(\delta)\,\beta\gamma}$

$I_5 = -\pi\,\mathbf{f_s}\,\mathbf{h}\cos(\delta-\varepsilon_2)\,\beta\gamma^2$ $\quad$ $I_8 = -\pi/2\,\mathbf{f_s}\,\mathbf{h}\sin(\delta-\varepsilon_2)\,\beta\gamma^2$

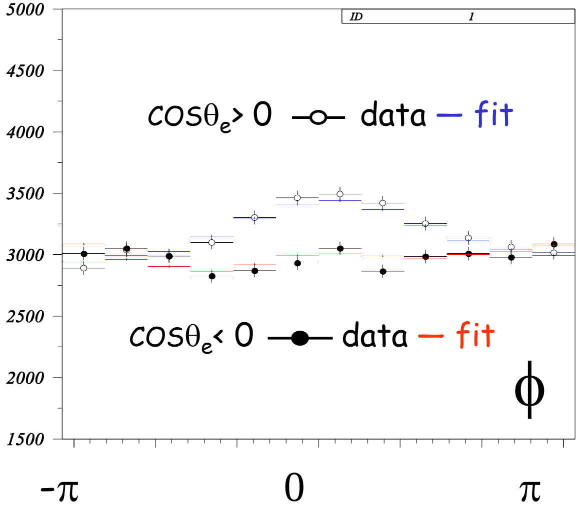
$I_6 = -8/3\,\mathbf{g}\,\mathbf{h}\cos(\varepsilon_2)\,\beta^2\gamma$ $\quad$ $I_9 = -8/3\,\mathbf{g}\,\mathbf{h}\sin(\varepsilon_2)\,\beta^2\gamma$

$\delta=\mathrm{atan}(I_7/2I_4)$ $\quad$ $\delta-\varepsilon_2=\mathrm{atan}(2I_8/I_5)$ $\quad$ $\varepsilon_2=\mathrm{atan}(2I_9/2I_6)$

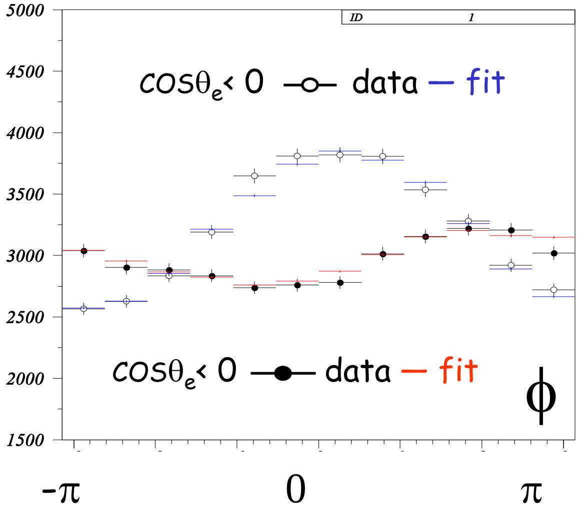


$\delta_{\pi\pi}$ signature in joint distribution $(\cos\theta_e, \phi)$

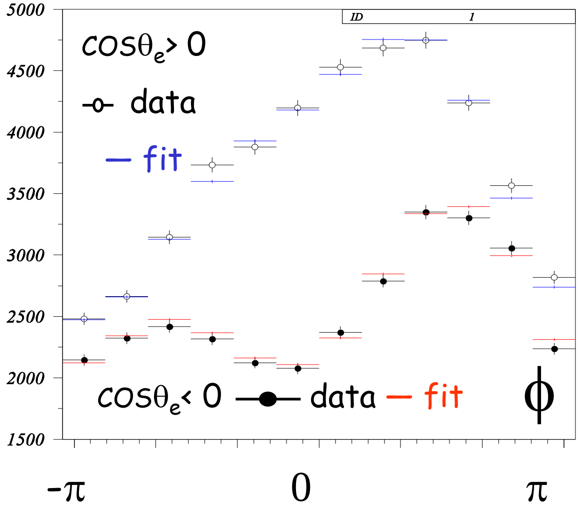
$m_{\pi\pi}$ bin 1



$m_{\pi\pi}$ bin 5



$m_{\pi\pi}$ bin 10



$\delta_{\pi\pi} \approx 0.03$ ⇨

asym($\cos\theta_e$) ≈0

asym(ϕ) ≈0

$\delta_{\pi\pi} \approx 0.15$ ⇨

asym($\cos\theta_e$) > 0

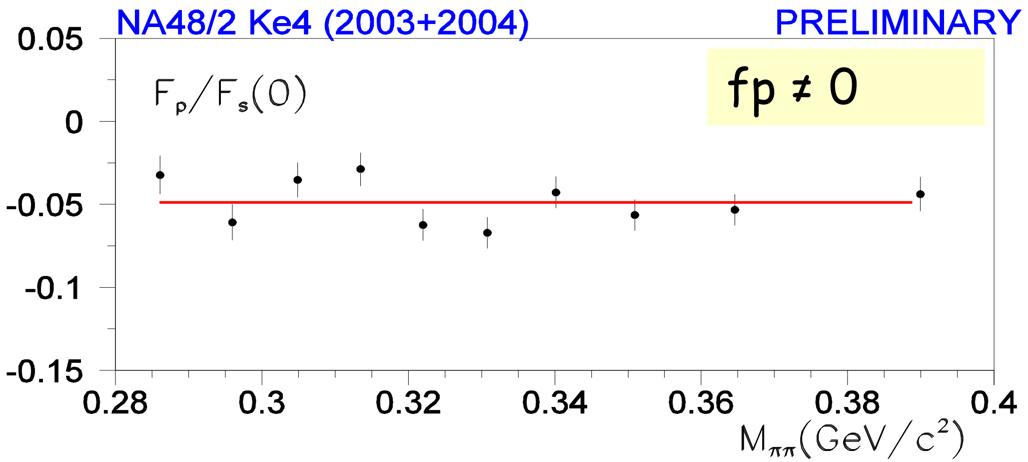
asym(ϕ) > 0

$\delta_{\pi\pi} \approx 0.30$ ⇨

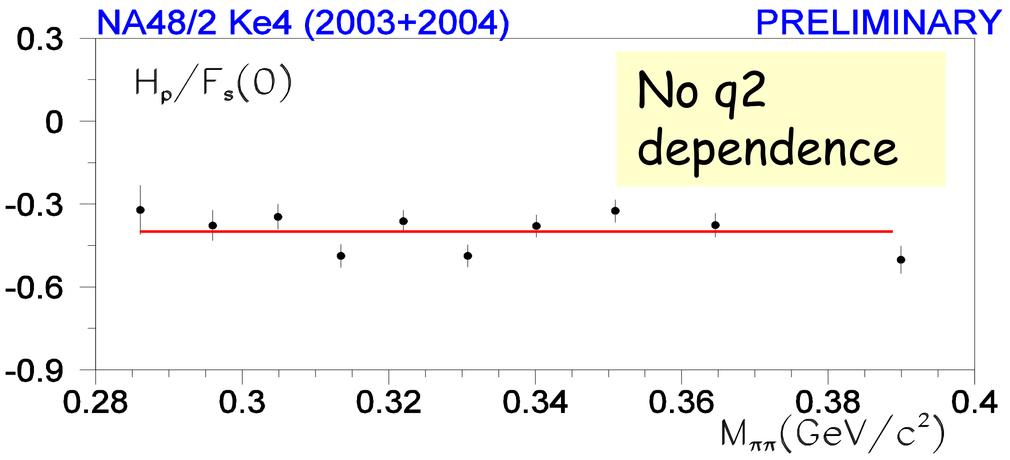
asym($\cos\theta_e$) >> 0

asym(ϕ) >>0

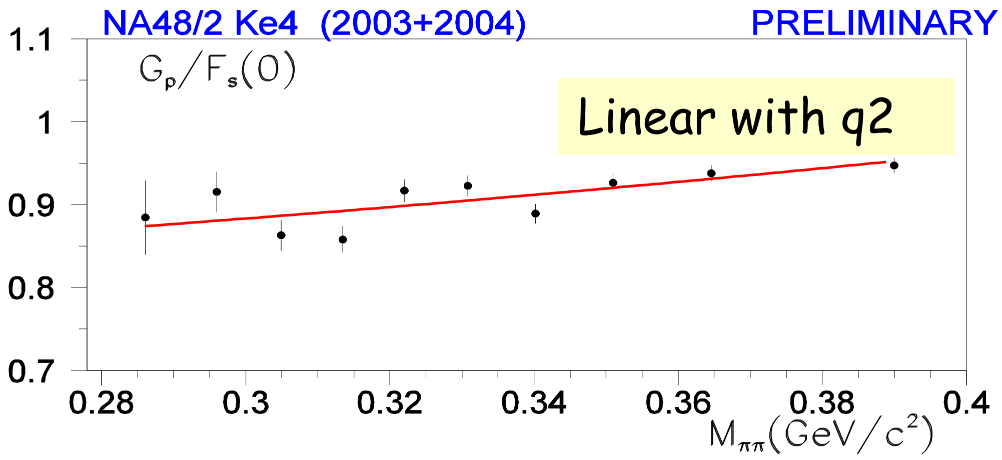
Getting Fp, Gp, Hp



fp ≠ 0



No q2 dependence



Linear with q2

Correlation

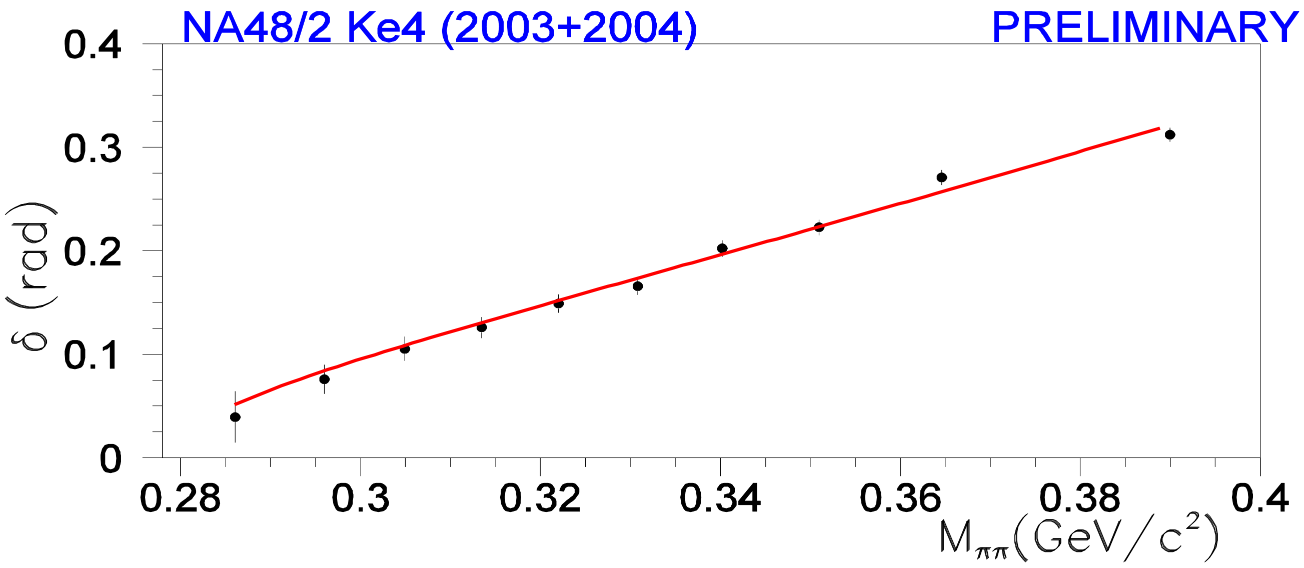
	g'_p
$g_p(0)$	-0.914

Ke4 charged decays : $\delta_{\pi\pi}$ and (a_0^0 ,a_0^2)

Extracting a_0^0 and a_0^2 from $\delta = \delta_0^0 - \delta_1^1$ variation requires

I=2 ππ data at $m_{pp} > 0.8$ + theoretical work

- numerical solutions of Roy equations (ACGL Phys. Rep.353 (2001), DFGS EPJ C24 (2002)) to connect δ with (a0, a2) ⇨ Universal Band where CL uses a relation $a_2 = f(a_0)$
- the χPT constrain band (CGL NPB603(2001)): reduced uncertainty(± 0.008)
- a 2-parameter fit can also be performed with 2 free parameters a_0^0 and a_0^2
- Isospin symetry breaking to be accounted for ("Bern" arxiv:hep-ph/0710.3048 by J. Gasser)



2 free parameters isospin corrections ON

Δa0 = ± 0.013 (stat)

Δa2 = ± 0.0084 (stat)

Ke4 charged decays : isospin corrections to δ

Using "Bern" corrections: 11 to 15 mrad over the fitted $M\pi\pi$ range

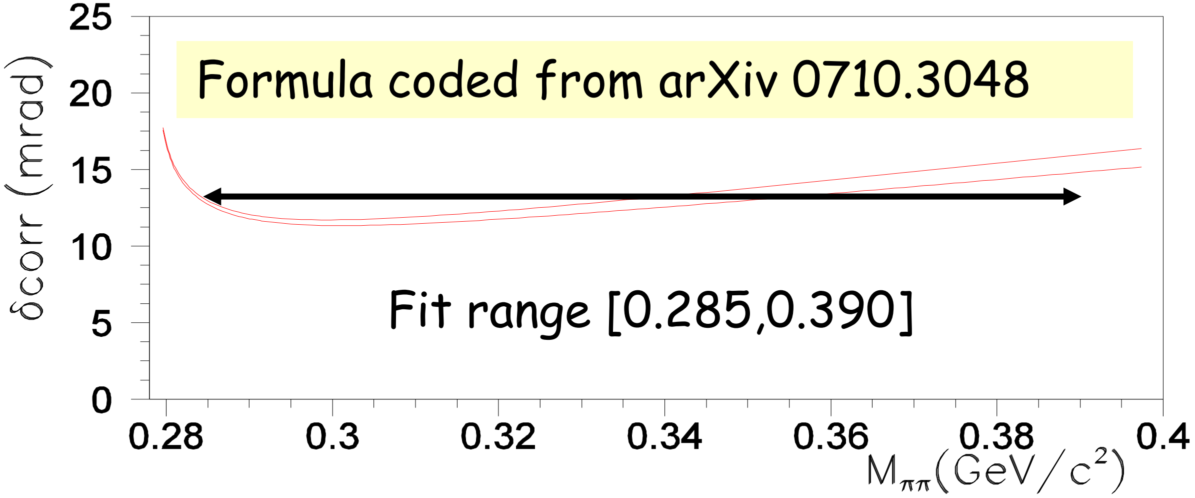
Uncertainty $R \pm \Delta R = 37 \pm 4$ ($R=m_s\text{-}m_{ud}/(m_d\text{-}m_u)$)

ΔR translates into $\Delta\delta$corr = at most ± 0.5 mrad @ 0.390 GeV/c^2

quoted as 'theoretical precision' (though marginal effect)

Well matched to the NA48/2 analysis where radiative and isospin symmetry breaking effects factorize:

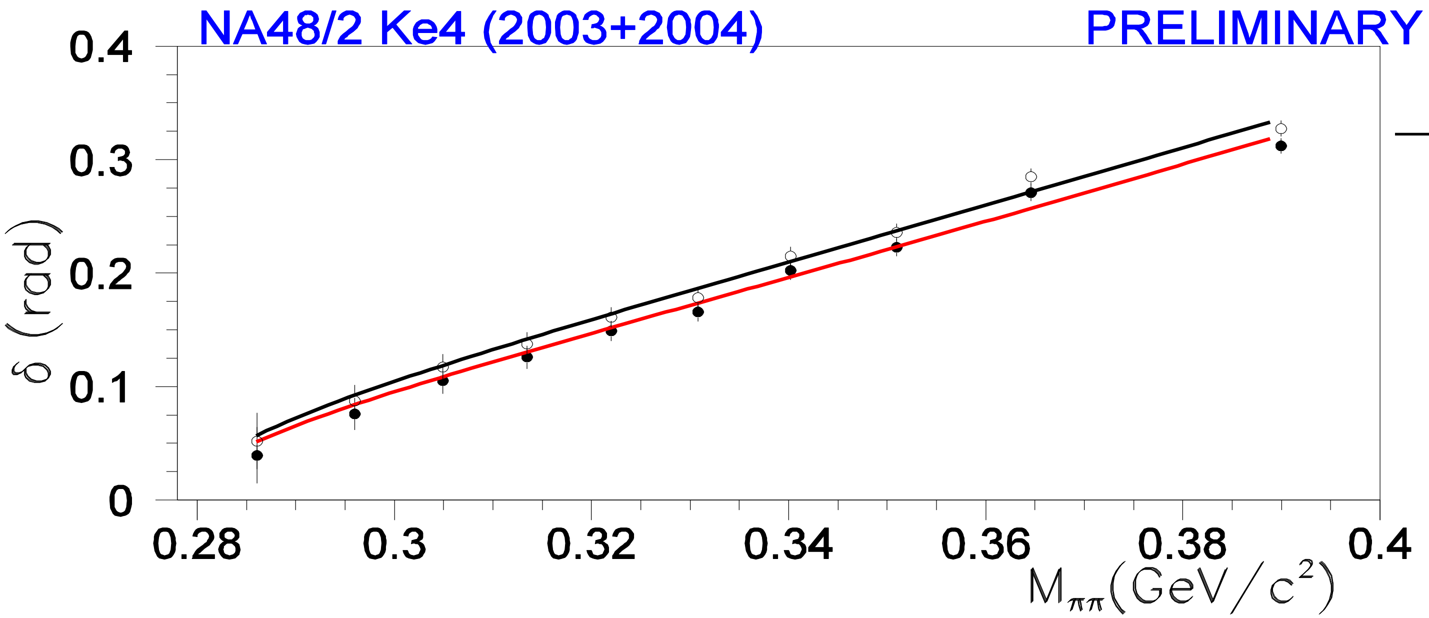
Gamow factor x PHOTOS generator x Isospin corrections



Ke4 charged decays : isospin corrections to δ

All Data K+ and K- combined=
○ = Isospin corr OFF
● = Isospin corr ON

Correction (~10mrad) is larger than the statistical error on each point above 0.3 GeV/c² (7-8 mrad)



2p fit	Isospin corr OFF	Isospin corr ON
a_0^0	0.244 ± 0.013	0.218 ± 0.013
a_0^2	-0.0385 ± 0.0084	-0.0457 ± 0.0084

Charged Ke4: results from 1 151 100 decays

relative Form Factors = FF/Fs(0)

- measured separately for K^+ and K^-,
- combined according to stat. errors,
- Fs obtained from bin to bin norm.
- Fp, Gp, Hp de-convoluted from observed Fs(q2,Se) variation .

Scattering length a_0 now measured with ~2% relative precision (χPT 1p fit),

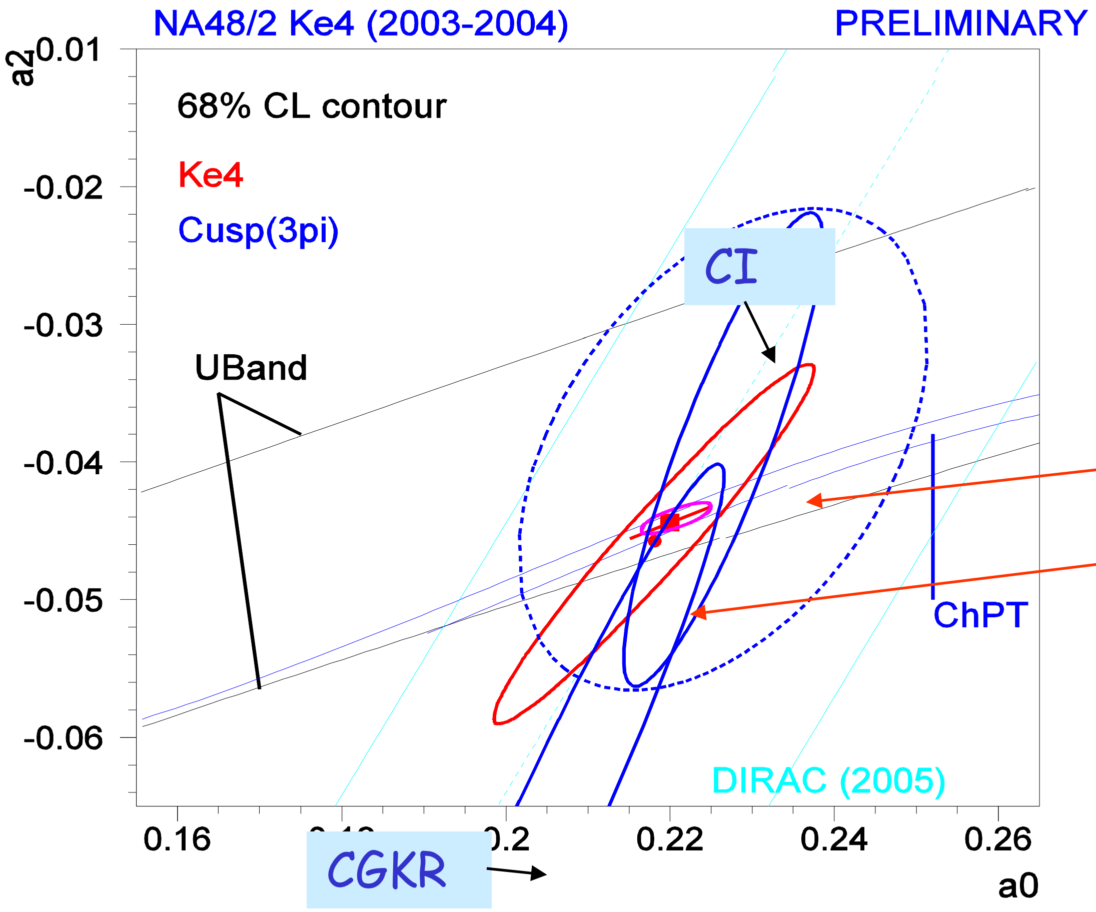
Improved precision for a_0 (~6%) and a_2 (~18%) in a 2-free parameter fit

Systematic uncertainty as for 2003 (conservative, will be revisited) but~0.5 x (stat) for a_0,a_2

value ± stat.

	2003 EPJC 54(2008)	2003 + 2004 preliminary
f'_s/f_s	0.172 ± 0.009	0.158 ± 0.007
f''_s/f_s	-0.090 ± 0.009	-0.078 ± 0.007
f'_e/f_s	0.081 ± 0.008	0.067 ± 0.006
f_p/f_s	-0.048 ±0.004	-0.049 ± 0.003
g_p/f_s	0.873 ± 0.013	0.869 ± 0.010
g'_p/f_s	0.081 ± 0.022	0.087 ± 0.017
h_p/f_s	-0.411± 0.019	-0.402 ±0.014
a_0 ChPT 1p fit $a_2=f(a_0)$	0.223 ±0.006 (-0.0437 ±0.0015)	**0.220 ± 0.005** (-0.0444 ±0.0011)
a_0 free	0.209 ± 0.016	**0.218 ± 0.013**
a_2 free	-0.0529±0.0105	**-0.0457±0.0084**

scattering lengths measurements by NA48



Two statistically independent measurements by NA48/2:

Cusp in K ($\pi^\pm\pi^0\ \pi^0$) : 1p fit within 2 models (CI, CGKR)

Ke4 with isospin corrections

- 2p fit using Roy equations
- 1p fit with χPT constraint

remarkable agreement with χPT

Precise χPT predictions : $a_2 = -0.0444 \pm 0.0008$ and $a_0 = 0.220 \pm 0.005$

Comparing with other measurements

Ke4 : isospin corrections to published $\delta_{\pi\pi}$ of all 3 experiments + 1p χPT fit

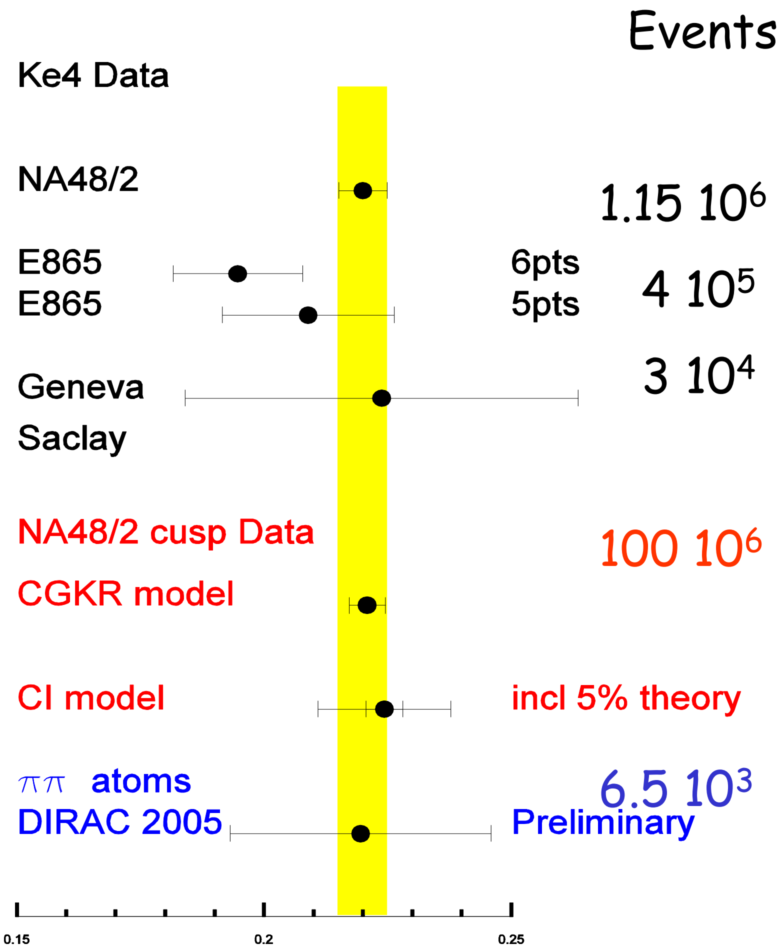
Note : E865 result dominated by highest energy data point, otherwise compatible

Cusp : use 1p χPT fit and 2 models

DIRAC : $|a_0 - a_2|$ non-symmetric errors from PLB619 (2005), use χPT constraint, still being revisited

Yellow band is χPT prediction

NA48/2 experimental precision now at the same level as theory!



a_0 from 1p ChPT fit, isospin corr ON

Epilogue

arXiv hep-ph/0212323v1 20 Dec 2002

$\pi\pi$ scattering: theory is ahead of experiment

H. Leutwyler

- 2003+04 K_{e4} and cusp data from NA48/2 has allowed to measure (a_0,a_2) with precision equal to χPT

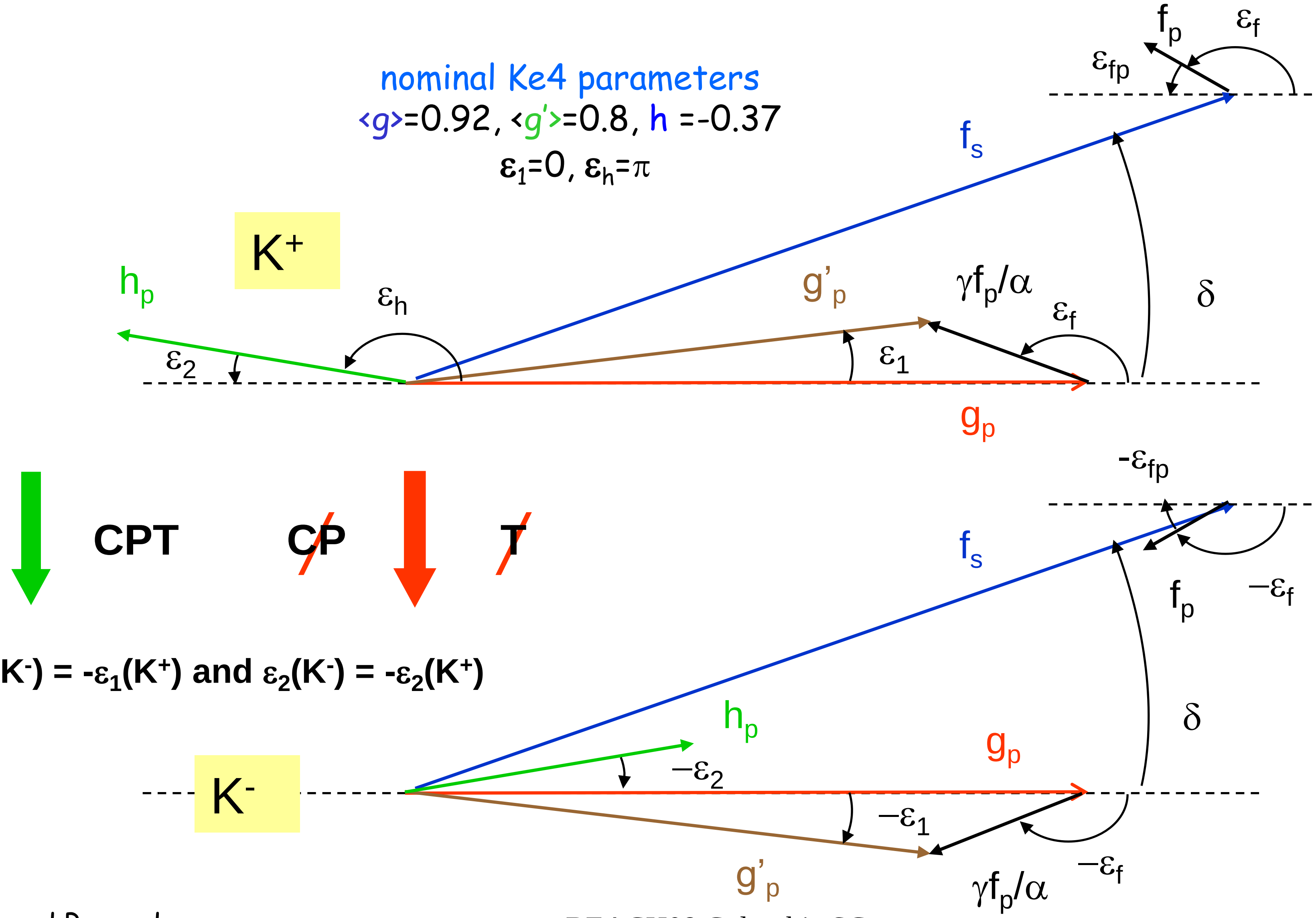
- data and theory agree remarkably well and it demonstrates that the pion mass is dominated (94%) by the quark condensate i.e. spontaneous breakdown of χPT (G. Colangelo, Garda workshop Feb. 2008)

$$M_\pi^2 = -\frac{<q\bar{q}>}{F_\pi^2}(m_u + m_d) + \mathcal{O}(m_q^2)$$

M. Gell-Mann, R. J. Oakes and B. Renner, Phys. Rev. 175 (1968) 2195

SPARES

K_{e4} amplitudes and phases with T and CP violation



nominal Ke4 parameters

$\langle g \rangle = 0.92$, $\langle g' \rangle = 0.8$, $h = -0.37$

$\varepsilon_1 = 0$, $\varepsilon_h = \pi$

$\varepsilon_1(K^-) = -\varepsilon_1(K^+)$ and $\varepsilon_2(K^-) = -\varepsilon_2(K^+)$